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THE "THIRD LETTER" OF NICHOLAS RHABDAS: AN AUTOGRAPH EASTER COMPUTUS

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Abstract

The article presents an edition, a translation, a technical commentary, and a thematic word index of the Easter Computus authored by the Byzantine mathematichian Nicholas Artabasdos Rhabdas. It is also shown that this work is preserved as an autograph.

Keywords: Easter Computi, Byzantine mathematician, autograph works

Resumen

El artículo presenta una edición, una traducción, un comentario técnico y un índice de palabras temáticas del Computus pascual escrito por el matemático bizantino Nicolás Artabasdos Rhabdas. También se muestra que esta obra se conserva como autógrafa.

***Metadata**: Computi* pascuales, matemático bizantino, obras autógrafas

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*THE "THIRD LETTER" OF NICHOLAS RHABDAS: AN AUTOGRAPH EASTER COMPUTUS**

Fabio Acerbi

That the early-14th century Byzantine scholar Nicholas Artabasdos Rhabdas (*PLP*, nr. 1437) wrote a fully-fledged Easter Computus—and not only the short thematic section included in his *Rechenbuch*, also referred to as the *Letter to Tzavoukhes*—is known since 1953. This piece of information is printed on page 81 of volume IX.2 of the *Catalogus Codicum Astrologorum Graecorum*, in the description of the manuscript Leeds, University Library, Brotherton Coll. MS 31/3. Seventy years later, this text is finally published in print.

Brotherton Coll. MS 31/3 is the last of a set of three manuscripts,[1] all containing treatises and extracts of astronomical and astrological argument. MS 31/1 comprises excerpts (accompanied by scholia)[2] from Ptolemy's *Almagest*, Books III and IV,[3] and

* I would like to thank Olivier Delouis for his assistance in Byzantine fiscal matters and for triggering this Rhabdas-Renaissance, Divna Manolova for drawing my attention to the fact that the Leeds manuscript contains a work by Rhabdas that is "unedited and [whose] addressee is unattested", Inmaculada Pérez Martín for her paleographic expertises, and Alessandra Petrocchi for her helpful comments on a first draft of this paper. After submitting this study I have come to know of the edition I. Σκούρα, "Μια ανέκδοτη επιστολή του Νικολάου Ραβδά για τους εκκλησιαστικούς λογαριασμούς", *Νεύσις* 27-28 (2019-20), 353-399 (but still in press at the time); this paper does not translate Rhabdas' Computus, does not contain any technical analysis, does not set a systematic comparison with parallel algorithms in other sources, does not present the Leeds manuscripts, and does not recognize Rhabdas' autography; for these reasons, and after some hesitation, I decided not to withdraw my paper. Readers will easily grasp the difference between the two approaches to Rhabdas' Computus.

1 The set of the three manuscripts is assigned the *Diktyon* number 37610, but only the third item in the set is described online at https://pinakes.irht.cnrs.fr/ under this number.

2 Inmaculada Pérez Martín (*per litteras*) has identified the copyist of MS 31/1 as Makarios (*RGK* III, nr. 398, referring to Vat. gr. 989 [Xenophon and other historians and tacticians, Nonnos; *Diktyon* 67620]; on this manuscript, see now I. Pérez Martín, "Enseignement et service impérial à l'époque paléologue : à propos de la formation des serviteurs des empereurs", *Travaux et Mémoires* 25 [2021], Appendix 1), a collaborator of Nicephoros Gregoras' who partly penned two manuscripts similar in content (identified in I. Pérez Martín, "Un escolio de Nicéforo Gregorás sobre el alma del mundo en el *Timeo* (*Vaticanus Graecus* 228)", *MHNH* 4 [2004], 197-219, at 209 and n. 45): Laur. Plut. 28.20 (astrological miscellany, description in R. Caballero Sánchez, "Historia del texto del *Comentario anónimo al* Tetrabiblos *de Tolomeo*", *MHNH* 13 [2013], 77-198, at 112-115; *Diktyon* 16201), ff. 1r-115v and 118r-267v, and Vat. gr. 1087 (*Diktyon* 67718), ff. 5r-27v (here Theodoros Metochites, *Astronomikē Stoicheiōsis*).

3 The extracts from the *Almagest* are found in ff. 1r-7r, *Alm*. III.1, 191.15-209.16 Heiberg; ff. 7r-12v, *Alm*. IV.1-3, 265.9-280.19 Heiberg.

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his entire *Hypotheses Planetarum*,[4] as well as extracts from Theon's commentary on the *Almagest*.[5] Short astronomical texts are also found in this manuscript.[6] MS 31/2 contains only the entire commentary of Stephanus of Alexandria on Ptolemy's *Handy Tables*, written by two hands (a collaborator of the main hand copied from the middle of the first line of f. 20v to the end of f. 21r).[7] MS 31/3 comprises astrological texts and Rhabdas' Computus on ff. 64r-69r (pp. 127-137). The contents of this manuscript are described in detail in *CCAG* IX.2[8] with the exception of the final table (ff. 70v-71r)

[4] This treatise is on ff. 13r-19r, which contain *Hyp*. 1-14, 70.3-104.23 Heiberg; this copy ends with the marginal note λείπει ἑξῆς στίχοι ια′ ὡς ἀπὸ τῶν προκειμένων; this is, in fact, the partly incomplete version of the treatise carried by Heiberg's second stemmatic family (*Claudii Ptolemaei opera quae exstant omnia*, *II*, *Opera astronomica minora*, ed. J.L. Heiberg, Lipsiae 1907, CLXIX-CLXXIV).

[5] These extracts are found on ff. 24v-27v, *in Alm*. I.4, 381.8-392.28 Rome; ff. 27v-33r, *in Alm*. IV.1, 946.16-967.2 Rome; ff. 33v-96v, VI, 273-311.13 (*des. mut*. τῆς ἡλιακῆς διαμέτρου) of the 1538 Basel edition. Recall that the Basel edition prints a Byzantine recension of Theon's commentary; however, MS 31/1 does not carry the text of the Byzantine recension.

[6] These short texts are found on ff. 19v-22r (see numbers II-XI and XIII, *des*. line 6, in A. Tihon, "Les scholies des Tables Faciles de Ptolémée", *Bulletin de l'Institut Historique Belge de Rome* 43 [1973], 49-110); ff. 22v-23r, a selection of the preliminary material found in some manuscripts of the *Almagest* (edition A. Jones, "Ptolemy's *Canobic Inscription* and Heliodorus' Observation Reports", *SCIAMVS* 6 [2005], 53-97); and ff. 23v-24r (see numbers I and XV in the cited paper by Tihon).

[7] This possibly important—and certainly one of the earliest—witness is not recorded in the recent edition of Stephanus' treatise, namely, J. Lempire, *Le commentaire astronomique aux* Tables Faciles *de Ptolémée attribué à Stéphanos d'Alexandrie. Tome I. Histoire du texte. Édition critique, traduction et commentaire (chapitres 1-16)* (Corpus des Astronomes Byzantins 11), Louvain-La-Neuve 2016. In MS 31/2, Stephanus' treatise (in 38 chapters; it belongs to Lempire's class II) occupies ff. 2r-76v. It is preceded by a pinax of the work (f. 1r; f. 1v is blank) and followed by the tables of the rising times of the zodiacal signs for the 7th *klima* and for Byzantium (f. 77r-v) and by an horoscope, that is, a list of the positions on the ecliptic of the seven planets and of the ascendant, for the day of Creation (as usual, only the zodiacal signs are marked) and for the foundation of Constantinople (f. 78r). On these horoscopes, edited in *Catalogus Codicum Astrologorum Graecorum*, I-XII, Bruxelles 1898-1953, *IX.2*, *Codices Britannicos*, *Pars altera*, ed. S. Weinstock, 1953, 176-178 (the editor did not rely on MS 31/2 as his source), see D. Pingree, "The Horoscope of Constantinople", in Y. Maeyama, W.G. Salzer (eds.), *ΠΡΙΣΜΑΤΑ. Naturwissenschaftsgeschichtliche Studien. Festschrift für Willy Hartner*, Wiesbaden 1977, 305-315. *Marginalia* of hands different from the main copyist are on f. 6r, note dated AM 6978 [= 1469] December 29, and on f. 76v, where one finds an undated note. The quire composition of MS 31/2, written on 26-31 lines per page, is 9×8, 1×8 – 1, the quire numbers are marked in the lower external corner of the first and of the last page of each quire (number ζ′ is wrongly located on f. 49r).

[8] *CCAG*, IX.2 (cit. n. 7), 78-81. The dimensions are mm 220×150, there are 25-31 lines per page. The hands are distributed as follows (I. Pérez Martín, *per litteras*). Hand 1: ff. 1r-6v,

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which, by employing Indo-Arabic numerals of the Western form,[9] lists all numbers from 11 to 100 along with one of their factorizations in two numbers noted as parts (as for instance $^1/_7$; three factors are indicated only for the numbers 96 and 98). In this table, prime numbers are marked by a special sign; they are further listed in a small table on f. 71v. I have not yet been able to understand the rationale behind the small table on f. 70r.

These three manuscripts were copied in the first half of the 14th century. In MS 31/3, the example given in the astrological text on f. 41v carries the date AM 6812 [= 1304] March 12[10]; this date agrees with the fact that several pages of the manuscript are written in an imitative script.[11] In MS 31/3, Rhabdas' Computus comes straight after the astrological collection, and is, in fact, an autograph of him, as a very recent finding confirms.[12] Rhabdas also penned most of MS 31/2; it is likely that he himself assembled this astronomical and astrological miscellany by adding his own transcriptions to pre-existing material. MS 31/3 was most likely available within Nicephoros Gregoras' entourage, since it has been very selectively corrected and annotated by Isaac Argyros, one of

8r, 11v-12r, 13r-v, 15r-23v, 24v-25r, 26r-27r, 28v-29r, 38r, 41r-42r15; hand 2, imitative: ff. 7r-v, 8v-11r, 12v, 14r-v, 24r, 25v, 27v-28r, 29v-37v, 38v-40v, 42r15-63v; hand 3 (Rhabdas): ff. 64r-69r. Marginalia and corrections on ff. 1r-2r, 4r-v, 13r, 18r, 41v, 63v, by Isaac Argyros. On f. 66v, lower margin, in red ink, numerals ͵ςϡοη κη σκ τκς. In Rhabdas' text, some brief parts have been re-written by a later hand to cover water damage.

9 According to Alessandra Petrocchi, whom I thank for the suggestion provided, some of these numeral-forms are common to those found in manuscripts written in medieval Latin and early Italo-Romance vernaculars and dating from the 13th and 14th centuries.

10 To denote dates, I adopt the astronomical convention era – year – month – day.

11 On imitative script in scientific manuscripts, see F. Acerbi, A. Gioffreda, "Manoscritti scientifici della prima età paleologa in scrittura arcaizzante", *Scripta* 12 (2019), 9-52. This style of writing was almost uniquely used in the period 1260-1310.

12 The finding involves a document of the Chilandar monastery dated 1323 and redacted by an imperial surveyor who signs himself as Nicholas Rhabdas; this document will be published in O. Delouis, M. Živojinović, *Actes de Chilandar. II. De 1320 à 1335* (Archives de l'Athos 24), Paris, forthcoming, nr. 90. Once the identification with our Rhabdas was confirmed by R. Estangüi Gómez, Estangüi Gómez himself, I. Pérez Martín and I conducted a cross-examination and were able to find Rhabdas' hand in the Leeds manuscripts, in his own (incomplete) square root table presented to Nicephoros Gregoras, now preserved in the manuscript Heidelberg, Universitätsbibliothek, Pal. gr. 129 (mainly 14th century; *Diktyon* 32460), ff. 11v-12r, and in Par. gr. 2650 (*Diktyon* 52285), ff. 147r-150v (ternion 147-152, the rest is blank apart from some monocondyla on f. 152r). The Paris manuscript exhibits another autograph work by Rhabdas, for ff. 147r-150v are the only extant witness of the grammar he composed for his own son Paul Artabasdos: see F. Acerbi, D. Manolova, I. Pérez Martín, "The Source of Nicholas Rhabdas' *Letter to Khatzykes*: An Anonymous Arithmetical Treatise in Vat. Barb. gr. 4", *Jahrbuch der Österreichischen Byzantinistik* 68 (2018), 1-37, n. 6 at 2-3.

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Gregoras' pupils and a prominent figure among the mathematically-minded scholars of the second half of the 14th century.[13]

The structure of this paper is as follows: in section 1, I introduce Rhabdas and his Computus, in section 2, I edit the Computus, translate it, and explain its content in a paraphrase and in a running commentary. In the paraphrase, I also present a symbolic transcription of the computational sections. The Appendix contains a thematic word index.

1. Introducing Rhabdas' Computus

Nicholas Artabasdos Rhabdas of Smyrna was a high-brow imperial functionary and scholar in Constantinople around 1320-42; he was connected with Nicephoros Gregoras and the circle of Maximus Planudes' pupils. His administrative role has been clarified by the same recent finding that has also led to the identification of his handwriting.[14] Rhabdas had a strong interest in mathematical matters; among other things, he wrote two logistic treatises in epistolary form: these are an arithmetic primer on the elementary operations with Indo-Arabic numerals (the so-called *Letter to Khatzykes*) and a *Rechenbuch* (the so-called *Letter to Tzavoukhes*). The latter also includes a short computistical section;[15] as in most Computi containing worked-out examples, this section also calculates the date of Easter for a year that is stated to be the current year, and the text can thus be dated to 1341. For the same reason, the autograph Computus in MS 31/3 can be dated to 1342.

In writing his Computus, Rhabdas adopted the same literary form he employed in writing the other two mathematical works mentioned above: a letter addressed to a friend.[16] It is plausible that Rhabdas' Computus (which I shall also call *Letter to Myrsiniotes*) is the last of the series of three mathematical letters we know he authored;

[13] On Argyros see A. Gioffreda, *Tra i libri di Isacco Argiro* (Transmissions 4), Berlin – Boston 2020.

[14] On Rhabdas' life and works (he wrote other texts) see the updated synthesis in Acerbi, Manolova, Pérez Martín (cit. n. 12), 2-6, and the new data collected in F. Acerbi, "A New Logistic Text by Nicholas Rhabdas", *Byzantion* 92 (2022). It goes without saying that Rhabdas' institutional role as a functionary of the fiscal administration fits remarkably well the contents of his *Letter to Tzavoukhes*.

[15] Editions of these two *Letters* are available in P. Tannery, "Notice sur les deux lettres arithmétiques de Nicolas Rhabdas", *Notices et extraits des manuscrits de la Bibliothèque Nationale* 32 (1886), 121-252, repr. Id., *Mémoires scientifiques*, IV, Toulouse – Paris 1920, 61-198, on pages 86-116 (*Letter to Khatzykes*) and 118-186 (*Letter to Tzavoukhes*). The computistical section in the *Letter to Tzavoukhes* is on pages 134.23-138.28.

[16] After Rhabdas, this format of scientific writing was also adopted by Isaac Argyros, in a short geometric metrological text (the *Letter to Kolybas*) and in his Easter Computus (the *Letter to Andronikos Oinaiotes*). See the discussion in Pérez Martín, "Enseignement" (cit. n. 2).

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what is certain is that the *Letter to Myrsiniotes* is later than the *Letter to Tzavoukhes*. Rhabdas' authorship is declared in the title of the *Letter to Myrsiniotes*, whose structure is identical to the structure of the title of the other two *Letters*. In our case, the addressee is an otherwise unknown Demetrius Myrsiniotes who, according to the title, was an elder, and particularly dear, friend of Rhabdas'. Were the title missing, Rhabdas' authorship would still be unquestionable because his autograph Computus begins with the same verbatim extract from the beginning of Diophantus' *Arithmetica* which opens the other two *Letters*. Moreover, in the final section of his autograph Computus Rhabdas reproduces a portion of his own brief Computus included in the *Letter to Tzavoukhes*; the reused passage provides an algorithm that allows one to calculate the date of Easter without having to compute that of Passover before. To sum up, Rhabdas expanded the computistical section of his own *Letter to Tzavoukhes* into a fully-fledged Computus.

Rhabdas' Computus presents standard features: it explains how to calculate the following items: indiction, solar, and lunar cycle years (sects. **2**-**5**), the "base" of the Moon (sect. **6**), the age of the Moon on a specific date (sect. **7**), the epacts of the Moon (sect. **8**), knowing its age, the visibility of the waxing and waning Moon (sect. **9**), the date of Passover (sect. **10**), the weekday on which Passover falls, and, consequently, the date of Easter (sect. **11**), what years are leap years (sect. **12**), the date of Meat-Fare Sunday (sect. **13**), the duration of Apostles' Fast (sect. **14**), and, finally, a paschalion Meat-Fare Sunday – Easter – Apostles' Fast, in this very order and, unlike the algorithm in sect. **11**, without using Passover (sect. **15**). Rhabdas' Computus is purely technical; only sects. **2** and **13** contain substantial discursive sequences, namely on the meaning and the origin of indiction (an excursus that tallies with Rhabdas' role in the Byzantine administration) the former, and on the disagreement over the date of Easter among some regional Christian churches the latter. All sections of this Computus present worked-out examples; apart from a single slip of pen, all given calculations are correct. However, as I shall point out in the next sections, the Computus contains some serious methodological mistakes, thereby suggesting that the material for which Rhabdas claims original authorship was, in fact, drawn from other sources. This is not surprising, as in his *Letter to Khatzykes* Rhabdas also silently appropriated an anonymous treatise written several decades earlier.[17] I have not been able so far to locate the Computus which has been Rhabdas' source for his *Letter to Myrsiniotes*.

2. Rhabdas' Computus: Edition, Translation, and Commentary

The manuscript containing Rhabdas' text presents only one copying mistake and four minor errors that can be attributed to distraction; there are also some corrections. In the

[17] This is shown in Acerbi, Manolova, Pérez Martin (cit. n. 12).

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edition which I present in this section I have retained the original accents of proclitics and enclitics. I have decided not to keep the original punctuation for the following reasons:[18] (1) in many occurrences, it is not possible to ascertain whether a point is marked by the author as upper or lower (and in some cases, even distinguishing between a comma and a point is impossible); (2) Rhabdas is not consistent with punctuating the algorithms: Computi use formulaic expressions and the author's inconsistency is sometimes self-evident; (3) studying Computi as a textual corpus involves comparing the algorithms they contain: uniformity in punctuation is therefore required. Other editorial conventions are: I have maintained adverbial expressions written in one single word as they appear; "aberrant" verb forms such as εὑρέθη (the augment is missing) are not corrected; numeral letters standing for integers are not marked by an apex; ordinals that in the text are given as numeral letters are written with a raised ending; according to the context, dates are treated as integers or as ordinals.

I have subdivided the text into thematic sections, most of which coincide with Rhabdas' paragraphs. The sequences that Rhabdas appropriated from Diophantus' *Arithmetica* (in sect. **1**) or that he reproduced from his own *Letter to Tzavoukhes* (in the title and in sect. **15**) are underlined. Each section presents the Greek text, its translation, preceded by a short title in italics, a paraphrase, and a commentary. The translation is faithful to the structure of the Greek text, especially within algorithms, where, however, I traslate the aorist tense by a present tense; readers will find in Rhabdas' Computus a fine specimen of immoderate—yet perfectly idiomatic—use of emphatic καί. A thematic word index is found in the Appendix. The paraphrase and the commentary are printed in reduced font size and are preceded by the titles ***Par*** and ***Comm***, respectively. My commentary to Rhabdas' work presents the context, clarifies some ambiguous points, and gives references to similar algorithms found in published Computi. The computational sequences are expressed in symbolic form.

Before presenting the text, some preliminary explanation of the determination of the date of Easter seems appropriate. The determination of the date of Easter in an assigned calendar year is traditionally reduced to finding the date and the weekday for that very year upon which the Jewish festival of Passover falls. This corresponds to the 14th day of a schematic lunar month and must occur on or straight after the Spring equinox,[19]

[18] I adopt the punctuation rules which I normally use in editing Greek and Byzantine mathematical texts and that are expounded in F. Acerbi, *The Logical Syntax of Greek Mathematics*, Heidelberg – New York 2021, sect. 1.4. In particular, such rules prescribe that consecutive steps of an algorithm are separated by an upper point; that a hiatus is marked by a full stop; that commas separate the principal clauses of a procedure and the result of a multiplication from the two factors.

[19] Passover falls on the 14th day of Nisan, the first month of Spring. On the early history of the Computus as a genre, see A. A. Mosshammer, *The Easter Computus and the Origins*

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whose date was fixed, as far as computistical matters are concerned, to March 21. Easter is the first Sunday after Passover; if Passover falls on Sunday, Easter is celebrated on the Sunday next thereafter.[20] Since Passover occurs on a fixed day of a specific lunar month, its date and the date of Easter vary from year to year. The dates of all other festivals in the annual Christian calendar which depend on Easter must vary with it, which explains why the Easter date must be calculated in advance. In order to determine it, it is essential to know—for the given year and possibly for a period of time—the beginning of each lunar month, that is, the date of the new Moon. This was ascertained by means of reasonably accurate approximations for the motions of the Sun and of the Moon, called "cycles"; in the case of the Moon, a cycle is a time interval after which the sequence of new Moons repeats itself on the same dates. In Computi, "Passover" is therefore the 14th day of a *schematic* lunar month in a lunisolar cycle (henceforth "lunar").[21] In the middle and late Byzantine period, a 19-year lunar cycle was almost unanimously adopted (see below for more details).

Once a cycle is adopted, all new Moons in it occur on fixed dates, which entails that the date of Passover in each year of the cycle is also fixed: a cycle uniquely determines a sequence of Passover dates,[22] which repeats with the periodicity of the cycle. Once the

of the Christian Era, Oxford 2008; still useful although poorly organized is V. Grumel, *La Chronologie* (Traité d'Études Byzantines 1), Paris 1958, 1-128. The Alexandrian Computus, from which the tradition of the Byzantine Computus stems, has been masterly reconstructed in O. Neugebauer, *Ethiopic Astronomy and Computus* (Sitzungberichte der Österreichischen Akademie der Wissenschaften 347), Wien 1979, and O. Neugebauer, *Abu Shaker's Chronography* (Sitzungberichte der Österreichischen Akademie der Wissenschaften 498), Wien 1988. See also the discussion in F. Acerbi, "Byzantine Easter Computi: An Overview with an Edition of *Anonymus* 892", *Jahrbuch der Österreichischen Byzantinistik* 71 (2021), where I edit what I have called *Anonymus* 892 and where the reader finds a list of the several *Anonymi* I shall cite in the following footnotes.

[20] To cite a source near to Rhabdas' times for instance, four "necessary conditions" (διορισμοί; I am pretty sure that Barlaam is alluding to the mathematical meaning of the term: see Acerbi, *The Logical Syntax* [cit. n. 18]. sect. 4.2.1) for the Easter date are emphasized by Barlaam in his Computus: A. Tihon, "Barlaam de Seminara. Traité Sur la date de Pâques", *Byzantion* 81 (2011), 362-411, at 376 (sect. 22).

[21] A lunar cycle is "lunisolar" because the occurrence of a new Moon depends on the position of the Sun. As lunar cycles are only approximations of the actual lunar motion and the length of the synodic month varies, the actual date of Passover and the computistical Passover may not always coincide; in Rhabdas' times this lack of synchronization amounted to about two days (see sect. **13**).

[22] Given the fact that a lunar month extends over 30 days and that the lunar cycle adopted in Byzantine Computi lasts 19 years, there are gaps in the sequence of the dates of Passover: see Rhabdas' table at the end of sect. **15**.

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date of Passover is known, one has to compute the weekday upon which it falls; the date of Easter is then easily determined. All the computations involved in the above-mentioned steps were formalized in standard discursive patterns which I refer to as "algorithms". Very simple algorithms compute the lunar cycle year of any assigned year in a given era (see sects. **2** and **5** in Rhabdas' Computus); knowing the lunar cycle year, the date of Passover can then be calculated (sect. **10**). Other algorithms compute the weekday of any assigned date in any given year (sects. **3**, **4**, and **11**). Combining these data, the date of Easter is easily found (again sect. **11**); from the date of Easter, the dates of all other movable feasts can be computed (sects. **13** and **14**). Any Computus, and Rhabdas' Computus in particular, consists of a collection of such algorithms; authors sometimes supply alternative algorithms (as for Rhabdas, see sect. **15**) and algorithms for computing other quantities which are differently relevant to the subject-matter (compare sects. **6**, **8**, and **12** to sects. **7** and **9**).

Title

|64r Μέθοδοι διάφοροι ἐκτεθεῖσαι παρὰ ἀριθμητικοῦ καὶ γεωμήτρου Νικολάου Σμυρναίου Ἀρταβάσδου τοῦ Ῥαβδᾶ αἰτήσει θύτου Δημητρίου τοῦ Μυρσινιώτου, περὶ τῆς ἰνδίκτου, τοῦ κύκλου τοῦ ἡλίου, τοῦ κύκλου τῆς σελήνης, τοῦ θεμελίου αὐτῆς, τῆς εὑρέσεως τῶν ἡμερῶν αὐτῆς, τῆς φαύσεως τῶν ὡρῶν αὐτῆς, ἔτι τὲ περὶ τῆς Ἀπόκρεω, τοῦ νομικοῦ Φασκαλίου, τῆς εὑρέσεως τῆς ἡμέρας τούτου καθ' ἣν γίνεται, τοῦ βισέξτου, τοῦ εὐσεβοῦς Πάσχα τῶν Χριστιανῶν καὶ τῆς ἐν τῷ θέρει γινομένης Νηστείας τῶν ἁγίων ἀποστόλων.

Several algorithms set out by the arithmetician and land-surveyor Nicholas Artabasdos Rhabdas of Smyrna, at the request of diviner Demetrius Myrsiniotes, about indiction, the cycle of the Sun, the cycle of the Moon, its base, the finding of its days, the hours of its visibility, and further, about Meat-Fare, Passover, the finding of the day on which this occurs, the leap year, the sacred Easter of Christians, and the Apostles' Fast that occurs in Summer.

1

Τὴν δήλωσιν τῶν παρά σου ζητηθέντων, ποθεινότατε καὶ γλυκύτατέ μοι πρεσβύτερε κῦρι Δημήτριε, γινώσκων σε σπουδαίως ἔχοντα καὶ καταλόγον μαθεῖν, ὀργανῶσαι τὴν μέθοδον ἐπειράθην, ἀρξάμενος ἀφ' ὧν συνέστηκε τὰ πράγματα θεμελίων, ὑποστῆσαι καὶ παραδοῦναι σοι τὴν τούτων εὕρεσίν τε καὶ μάθησιν. ἴσως μὲν οὖν δοκεῖ τὸ πρᾶγμα δυσχερέστερον, ἐπειδὴ μήπω γνώριμόν ἐστι – δυσέλπιστοι γάρ εἰσι πρὸς κατόρθωσίν αἱ τῶν ἀρχομένων ψυχαί – ὅμως δ' εὐκατάληπτόν σοι γενήσεται διά τε τὴν σὴν προθυμίαν καὶ τὴν ἐμὴν ἀπόδειξιν· ταχεῖα γὰρ εἰς μάθησιν ἐπιθυμία προσλαβοῦσα διδαχήν.

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Introduction

As I know that the clarification of the things you have been seeking is something you, my bitterly longed for and dearest sir Demetrius, earnestly strives to learn on a rational basis, I tried, beginning from the bases on which the subject-matter rests, to give shape to a systematic exposition in order to lay down and to transmit to you the way of both finding and learning these things. Then, the subject-matter may seem particularly difficult since it is not yet familiar—for the beginners hardly feel hopeful for a successful accomplishment—still, it will become easily apprehensible for you thanks both to your eagerness and to my rigorous exposition, for eagerness enriched by teaching runs fast towards learning.

Comm. The introductory section is almost entirely drawn verbatim from Diophantus' *Arithmetica*.[23] The same extract also opens the *Letter to Tzavoukhes* and, with the exception of the length of the portions excerpted, the *Letter to Khatzykes*.[24] Apparently, Rhabdas considered using this quote in his grammatical *Letter to Paul Artabasdos* unsuitable. Nothing else is known about Demetrius Myrsiniotes (whose appellation θύτης "diviner" is perplexing); however, two Myrsiniotes are recorded as *PLP*, nr. 92694 and 92695.

2

Καὶ πρῶτον γε λεκτέον περὶ ἰνδίκτου· τί ἐστιν ἴνδικτος, καὶ τί τὸ ταύτης ὄνομα δηλοῖ, καὶ πόθεν ἤρξατο καὶ παρὰ τίνος. τὸ τῆς ἰνδίκτου ὄνομα λατινικόν ἐστι, σημαίνει δὲ δύο, τήν τε ἀρχὴν καὶ τὴν ἐπινέμησιν. τὴν μὲν ἀρχὴν διότι τοῦ παντὸς κόσμου κατὰ τὴν εἰς τὸν Κριὸν τοῦ ἡλίου εἰσέλευσιν τόδε τὸ πᾶν ἐδημιουργήθη καὶ ἐξ οὐκ ὄντων παρήχθη παρὰ τοῦ τῶν ὅλων ἀριστοτέχνου θεοῦ, ὃ δὴ ζῴδιον μῆνα Μάρτιον ἡμεῖς ὀνομάζομεν, μῆνα δὲ λέγομεν τούτων ἕκαστον ἐκ τοῦ τῆς σελήνης ὀνόματος – μὴν γὰρ ἡ σελήνη λέγεται, καὶ ἀπὸ ταύτης καὶ οἱ τοῦ ἡλίου μῆνες τὴν προσηγορίαν ἐσχήκασι –, καὶ ὅτι ὁ μὲν ἥλιος ἐφ' ἑκάστῳ μηνὶ ἢ καὶ πλείονι ἢ καὶ ἐλάττονι διαστήματι ἓν διέρχεται ζῴδιον, ἐπειδὴ οὐκ εἰς ἴσα τμήματα τέμνει τὰ δωδεκατημόρια ἀλλ' εἰς ἄνισα διὰ τὸ τὸν τούτου κύκλον ἔκκεντρον εἶναι πρὸς τὸν ζῳδιακόν, ἡ δὲ σελήνη καθ' ἕκαστον μῆνα τὰ ιβ διέρχεται ζῴδια, καὶ διὰ τοῦτο ἐπεκράτησεν ἡ τοῦ χρόνου ἀρχὴ γίνεσθαι (καθὼς ὁ θεσπέσιος καὶ μέγας προφήτης Μωυσῆς ἡμῖν παραδίδωσι) κατὰ τὴν ἀρχὴν τοῦ Μαρτίου μηνός. ἐπεκράτησε δὲ οὕτως εἶναι καὶ λέγεσθαι χρόνους ,ευξ· ἐν δὲ τῷ τετάρτῳ ἔτει τῆς Αὐγούστου καίσαρος βασιλείας, παρὰ τοῦ τοιούτου καίσαρος ἡ τοῦ χρόνου μετετέθη ἀρχὴ κατὰ τὴν πρώτην

[23] P. Tannery (ed.), *Diophanti Alexandrini opera omnia cum Graeciis commentariis*, I-II, Lipsiae 1893-95, I, 2.3-13

[24] Compare Tannery, "Notice" (cit. n. 15), 86.6-15 and 118.3-10, and Tannery, *Diophanti opera* (cit. n. 23), I, 2.3-17 and 2.3-13, respectively.

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τοῦ Σεπτεμβρίου μηνὸς δι᾽ αἰτίαν τοιαύτην, ὅτι οἱ ἐτήσιοι φόροι καὶ οἱ δασμοὶ καὶ τὰ τέλη καὶ αἱ διανομαὶ κατὰ τοῦτον ἐγίνοντο τὸν καιρόν, μετὰ τὴν τῶν καρπῶν δηλονότι συγκομιδήν. ἐκάλεσαν οὖν Λατῖνοι τὴν τοῦ χρόνου ἀρχὴν καὶ ἴνδικτον καὶ ἰνδικτιῶνα καὶ ἐπινέμησιν διὰ τὸ γίνεσθαι κατὰ τὸν τοιοῦτον καιρὸν τὰς τῶν χρημάτων δόσεις καὶ διανομάς· εἰς πεντεκαίδεκα δ᾽ ἐνιαυτοὺς τὴν ἴνδικτον ἐθέσπισεν ἀνιέναι, καὶ ἔκτοτε πάλιν λαμβάνειν ἀρχήν, ἢ διὰ τὸ μέχρι τοσούτου τὰς ἐπισκέψεις γίνεσθαι τῶν πραγμάτων ἢ διὰ τὸ τὰς δόσεις ἐπὶ τοσοῦτον κατέχεσθαι καὶ πάλιν ἀνακάμπτειν ταύτας πρὸς τοὺς δεσπότας. ἔλαβε |$_{64v}$ τοίνυν τὴν ἀρχὴν ἡ ἴνδικτος κατὰ τὸ ‚ευξ ἔτος.

Ὅτε οὖν βούλει τὴν ἐνισταμένην εἰδέναι ἴνδικτον, ἀνάλυσον ἐπὶ τὸν ιε τὰ εὑρισκόμενα ἔτη ἀπὸ τοῦ ‚ευξ ἔτους μέχρι τοῦ νῦν ἐνισταμένου ἀπὸ κτίσεως κόσμου ‚ςων ἔτους· καὶ εἰσὶ ‚ατϙ. λέγε οὖν οὕτως· ιε ϙ, ‚ατν· ἐναπελείφθησαν καὶ μ. καὶ πάλιν εἰπέ· ιε β, λ· ἔμεινάν σοι καὶ ι.

Καὶ ἀπὸ τῆς ὅλης δὲ συναγωγῆς τῶν ὅλων ἀπ᾽ ἀρχῆς τοῦ κόσμου ἐτῶν (ἤγουν τῶν ‚ςων) εἰ βούλει τὴν ἴνδικτον εὑρεῖν, ἀνάλυσον ἐπὶ τὸν ιε τὰ ‚ςων, καὶ τὰ κάτωθεν εὑρισκόμενα τῶν ιε ἡ ἐνισταμένη ὑπάρχει ἴνδικτος. λέγε δὲ οὕτως· ιε υ, ‚ς· ιε ν, ψν· ιε ς, ϙ· λοιπὰ ι. καὶ ἔστιν ὁ τῆς ἰνδίκτου κύκλος ι, ὡς καὶ ἀνωτέρω δεδήλωται.

Ἐγὼ δὲ καὶ ἄλλην ἐφεῦρον συντομοτέραν μέθοδον ῥᾴστην[a] οὖσαν πρὸς εὕρεσιν διὰ τοὺς ἐνδεῶς πρὸς τὴν τῶν ἀριθμῶν ἔχοντας δύναμιν, ἣν δὴ καὶ προσθῆσαι οὐκ ὤκνησα τῷ παρόντι πονήματι, ἥ δέ ἐστιν αὕτη. κράτησον ἀπὸ τῶν ‚ςων ἐτῶν τὰ ν μόνα, καὶ τούτοις πρόσθες καὶ ε, ἅτινα δηλονότι καὶ περιττεύουσιν ἐκ τῶν ‚ςω ἀφαιρουμένων τῶν πεντεκαιδεκάδων πασῶν· καὶ γίνονται νε ἐκ τούτων· σκόπει ποσάκις δύνῃ τὸν \ιε/ ἀριθμὸν ἐκβαλεῖν, καὶ εὑρήσεις πάντως τρισσάκις· τρὶς γὰρ τὰ ιε, με, ὧν ἀφαιρουμένων ἐκ τῶν νε καταλιμπάνονται ι, ἴσα ὄντα καὶ τοῖς πρότερον εὑρεθεῖσιν ἐκ τῶν ἄλλων μεθόδων. καὶ ταῦτα μὲν περὶ τῆς εὑρέσεως καὶ καταλήψεως τῆς ἰνδίκτου.

[a] $^{\beta}$μέθοδον $^{\alpha}$συντομοτέραν $^{\gamma}$ῥᾴστην

Indiction and indiction cycle

And of course, one must first speak about the indiction: what is indiction, and what signifies its name, and from where it took its origin, and by whom. The name “indiction” is a Latin one, and signifies two things, “beginning” and “apportioning”. <It signifies> “beginning” because the whole Cosmos when the Sun was entering Aries,[25] this whole was created and brought in from non-being by God, best-artificer of the whole, which sign we do call “month of March”, and we call each of these “month” from the name of the Moon—for the Moon is also called “month”, and the months of the Sun have also got

[25] This marked anacoluthon and the long-winded sentence that includes it show that Rhabdas is partly improvising his Computus. We shall find other syntactic incongruities in the text.

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their denominations from this—and because the Sun traverses one sign each month, or in a greater or even in a lesser interval, since it does not cut the signs into equal segments but into unequal ones because its circle is eccentric with respect to the ecliptic, whereas the Moon traverses the 12 signs each month, and for this reason (exactly as the divine and great prophet Moses hands down to us) the beginning of the year was retained to occur at the beginning of the month of March. It was so retained to be and said for 5460 years; however, in the fourth year of the reign of the emperor Augustus, the beginning of the year was shifted to the first of the month of September by so great an emperor, for the following reason: the annual tributes, taxes, dues, and regulations were gathered on this occasion,[26] as is clear, after the harvest. Then, the Latins called the beginning of the year both "indiction" and "apportioning" because taxes and regulations on properties are gathered on such an occasion; <Augustus> decreed that the indiction should go up to fifteen years and thereafter take again its beginning, either because, after such a period, tax censuses are revised or because, after such a <period>, taxes are collected and again returned back to the ruler. Now then, indiction took its beginning in year 5460.

Then, whenever you wish to know the present indiction, resolve out into 15 the years found from year 5460 up to the now-present year 6850 from the foundation of the world; and they are 1390. Then, say as follows: 15 <times> 90, 1350; 40 are also left out. And again, say: 15 <times> 2, 30; there also remain 10 for you.

And if you wish to find the indiction starting from the whole gathering of all the years from the beginning of the world (namely, 6850), resolve 6850 out into 15, and that which is found down from 15 turns out to be the present indiction. Say as follows: 15 <times> 400, 6000; 15 <times> 50, 750; 15 <times> 6, 90; 10 as a remainder. And the cycle of the indiction is the 10th, as clarified above too.

I have also found another, more concise algorithm, which, for the purpose of finding, is most readily used by those who are insufficiently skilled in numbers, and which, of course, I did not hesitate to add to the present work too, and which is as follows. Keep only 50 from the 6850 years, and add 5 to these too, which, as is clear, also remain over from 6800 once all pentadecads are removed; and they yield 55 from these; consider how many times you can cast number 15 aside, and you will always find tree times, for thrice 15, 45, which once removed from 55, 10 are left out, which are also equal to those found above by means of the other algorithms. And these things about finding and apprehending the indiction.

26 As O. Delouis suggested *per litteras*, these terms "désignent, ainsi que d'autres, les impôts de manière générique" and "relèvent plutôt de l'accumulation rhétorique". Accordingly, I have translated these four terms with four generic English terms of similar meaning. On the fiscal terminology in the Palaiologan period, see A. Kontogiannopoulou, "La fiscalité à Byzance sous les Paléologues (13e-15e siècles)", *Revue des Études Byzantines* 67 (2009), 5-57.

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Par. The meaning and origin of "indiction" (ἴνδικτος) is as follows: the meaning of indiction (a "Latin term") is "beginning" because, as the prophet Moses also attests, Creation took place when the Sun was entering Aries and the beginning of time was thereby set to this specific yearly calendar date (an explanation of the meaning and etymology of the noun μήν, "month" but also "Moon", is also provided), and because the Sun traverses each sign of the zodiac in different times because of the eccentricity of its orbit, whereas each month the Moon traverses all 12 signs; the meaning of indiction is also "apportioning" (ἐπινέμησις). A historical outline of indiction follows: it was introduced by Augustus in AM 5460, which was the fourth year of his reign, by shifting the beginning of the year (and hence of indiction) from the Spring equinox to September 1, when harvest season ends and annual taxes are accordingly collected. The designation "apportioning" derives from what has been explained. The indiction cycle lasts 15 years either because, after such a period, tax censuses are revised or because, in the same period, taxes are collected and returned to the ruler.

The algorithm for finding the indiction cycle year i of an assigned year y in the Byzantine world era is:[27]

$(y) \rightarrow y - 5460 \rightarrow (y - 5460) \bmod 15 = i$.

A computation is carried out for current year AM 6850 [= 1341/2], and it yields $y = 6850 \rightarrow i = 10$.

By using directly the world era, the algorithm is:

$(y) \rightarrow y \bmod 15 = i$.

[27] My notation is as follows: I use the signs 1_J, 1_F, 1_M, 1_A, 30_S, etc., for January 1, February 1, March 1, April 1, September 30, etc. The crucial operation in a Computus is finding the remainder of the division of a number x by a number n. In modern terms, this is the "modulo" reduction, whose sign is "$x \bmod n$". The sign "$x \equiv y \pmod{n}$" (read "x is congruent to y modulo n") signifies that numbers x and y, once divided by n, yield the same remainder. The sign $[\![x]\!]$ denotes the floor (integral part) of number x, namely, the nearest integer (0 included) less than or equal to x. A sign like $\sum_{k=J}^{X-1} n_k$ stands for the sum of a sequence of numbers n_k in which the index k runs from J to $X - 1$: this is the sum of the lengths expressed in days of the months from J(anuary) up to an assigned month, denoted by $X - 1$. The sum gives null values when the assigned month is January. Counting, for instance the days from 1_M, is denoted by the sign "–:". The symbolic transcriptions I use in my paraphrase and in my commentary are intended to represent the computational flow faithfully: the initial input is the assumed quantity and this is given within parentheses: (y); a self-contained step of the transcription formalizes a complete clause of the formulated algorithm (note that this means that several operations may be represented); steps in which the output-input chain is not interrupted are linked by an arrow →; the operands of a given step are usually written in the same order as they are found in the text; the sign | separates independent steps that follow one and the same step (that is, a branching has occurred); a full stop indicates an algorithmic hiatus or the end of an algorithmic branch; levels of brackets go iteratively from parentheses to braces; the final output is preceded by the sign =.

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A computation is carried out for current year AM 6850 [= 1341/2], and it yields $y = 6850 \rightarrow i = 10$.

A more concise and very easy algorithm, which Rhabdas claims to be his own discovery and which he does not hesitate to include in his Computus, is:

$(y) \rightarrow y - 6800 \rightarrow (y - 6800) + 5 \rightarrow [(y - 6800) + 5] \bmod 15 = i$.

This algorithm relies on the fact that $5 \equiv 6800 \pmod{15}$. A computation is carried out for current year AM 6850 [= 1341/2], and it yields $y = 6850 \rightarrow i = 10$.

Comm. The indiction is a 15-year cycle introduced in the late Roman empire for taxation purposes. There are several regional variants of the indiction cycle, and its initial history is complex;[28] AD 312/3 is year 1 of the most current indiction cycle. The indiction cycle and the Byzantine world era are synchronized:[29] year 1 of the Byzantine world era is also year 1 of the indiction cycle; moreover, both the Byzantine civil year and the indiction year begin on September 1. Thus, computing the indiction cycle year starting from a year *y* of the Byzantine world era amounts to finding the remainder after subtracting 15 units from *y* as many times as possible; the related algorithm is the second given by Rhabdas. He also makes indiction to start at AM 5460; of course, since $5460 \equiv 0 \pmod{15}$, we should take this statement to mean that AM 5461 is indiction year 1. That $5460 \equiv 0 \pmod{15}$ also proves that the first algorithm given by Rhandas is correct.

In the third algorithm, as well as in the algorithms for solar and lunar cycle years, the nearest end-of-century year is removed from a world era year before the modulo reduction is carried out. Since $6800 \equiv 5 \pmod{15}$, $6800 \equiv 24 \pmod{28}$, and $6800 \equiv 17 \pmod{19}$, the algorithms for the indiction, solar, and lunar cycles entail not only the shift $y \rightarrow y - 6800$, but adding the above numbers as parameters to compensate for the shift. Below is a table of the values of *i*, *s*, and *m* for the end-of-century years that are relevant to Byzantine Computi[30]:

[28] A detailed study is by S. Bagnall, K.A. Worp, *Chronological Systems of Byzantine Egypt*, 2nd ed., Leiden – Boston 2004; Grumel, *La Chronologie* (cit. n. 19), 192-206 provides a brief account and explains the regional variants. See also the account in Mosshammer, *The Easter Computus* (cit. n. 19), 20-24.

[29] An Era is a non-cyclic count of calendar years starting from a given year 1, called "epoch". The epoch of the Byzantine world era (τὰ ἀπὸ κτίσεως κόσμου ἔτη "the years from the foundation of the world") is BC 5509 September 1, which falls on a Saturday; years are Julian years. On eras, see the synopsis in Grumel, *La Chronologie* (cit. n. 19), 207-226 and 279-296; see also O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin – Heidelberg – New York 1975, 1143 *s.v.*, and especially 1064-1067 and 1074-1076 (with bibliography), and the dedicated sections in Neugebauer, *Ethiopic Astronomy* (cit. n. 19), and Neugebauer, *Abu Shaker's* (cit. n. 19).

[30] Other Computi in which end-of-century years are subtracted—despite his claim for originality, many of them precede Rhabdas' times—are *Anonymus* 892, sect. 7; *Anonymus* 1092A, sect. 1, and 1092B, sect. 1 (both for indiction only), edited in F. P. Karnthaler, "Die chronologischen Abhandlungen des Laurent. Gr. Plut. 57, Cod. 42. 154-162v", *Byzantinisch-neugriechische*

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	6200	6300	6400	6500	6600	6700	6800	6900
i	5	0	10	5	0	10	5	0
s	12	0	16	4	20	8	24	12
m	6	11	16	2	7	12	17	3

3

Περὶ δὲ κύκλου ἡλίου καὶ σελήνης ῥητέον ὧδε. ὁ τοῦ ἡλίου κύκλος ἄρχεται μὲν ἀπὸ τῆς πρώτης τοῦ Ὀκτωβρίου μηνός, ἀνέρχεται δὲ εἰς χρόνους κη, καὶ πάλιν λαμβάνει ἀρχήν. τί οὖν ἐστιν ὃ κύκλον[a] ἡλίου λέγομεν· καὶ διὰ τί ὁ Ὀκτώβριος μὴν θεμέλιος τῶν τοῦ ἡλίου λέγεται κύκλων, καὶ κατὰ τίνα λόγον οἱ τοῦ ἡλίου κύκλοι κη εἰσὶ καὶ οὐ πλείονες οὐδ᾿ ἐλάσσονες, ὁμοίως καὶ διὰ τίνα λόγον ὁ Ἰαννουάριος μὴν θεμέλιος τῶν τῆς σελήνης κύκλων λέγεται, καὶ διὰ τί οἱ τῆς σελήνης κύκλοι ιθ εἰσί, οὐκ εὔκαιρον νῦν λέγειν ἐν τῷ παρόντι τοὺς συντομίᾳ χρωμένους.

[a] lege ὁ κύκλος

Solar and lunar cycles

On the cycle of the Sun and of the Moon one must say in the following way. The cycle of the Sun begins on the first of the month of October, reaches to 28 years, and takes again its beginning. Then, we say what is the cycle of the Sun; and why the month of October is called "base" of the cycles of the Sun, and for what reason the cycles of the Sun are 28, and not more nor less, similarly, for what reason the month of January is also called "base" of the cycles of the Moon, and why the cycles of the Moon are 19, it is out of place to say now, for in the present <exposition> we aim at conciseness.

Comm. Solar cycles of equal length exhibit the same sequence of pairings between dates and weekdays. As Julian years model the tropical year of $365^1/_4$ days (and therefore an intercalary day is added every fourth year, see sect. **12**), the number of weekdays and 4 are prime to one another, and neither 365 or 366 are multiples of 7, therefore the shortest solar cycle consists of 7×4 = 28 Julian years. Once an assigned sequence of calendar years is divided into consecutive

Jahrbücher 10 (1933), 1-64, at 5.1-3 and 8.136-138, respectively; *Anonymus* 1247, sects. 2, 6, 8, edited in O. Schissel, "Chronologischer Traktat des XII. Jahrhunderts", in Εἰς μνήμην Σπ. Λάμπρου, Ἀθῆναι 1935, 105-110, at 106-107; sects. 2-4 of the unpublished *Anonymus* 1256 in Vat. Pal. gr. 367 (*Diktyon* 66099), ff. 85r-88r; *Anonymus* 1273, sect. 3, edited in edited in F. Buchegger, "Wiener griechische Chronologie von 1273", *Byzantinisch-neugriechische Jahrbücher* 11 (1934-35) 25-54, at 29.19-27; Matthew Blastares (dated 1335), edited in G. Rhalles, M. Potles, Σύνταγμα τῶν θείων καὶ ἱερῶν κανόνων κατὰ στοιχεῖον, VI, Ἀθῆναι 1859, 414-416; *Anonymus* 1350, sects. 1-3, in O. Schlachter, *Wiener griechische Chronologie von 1350*, Diss. Graz 1934, 5.3-6.14; Isaac Argyros' Computus (dated 1372), sects. 3 and 6, in *PG* XIX, 1284-1285 and 1292; *Anonymus* 1377, sects. 1-2, 4, in *PG* XIX, 1317 and 1321; *Anonymus* 1379, in *PG* XIX, 1329.

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solar cycles, and thanks to the defining property of solar cycles, an algorithm able to determine the weekday of an assigned date within a solar cycle also calculates it for any year in the assigned sequence. Synchronizing solar cycles with the current era works out the same problem for any given calendar year. As is usual in Computi, Rhabdas uses κύκλος to name both the 28-year solar “cycle” and a solar “cycle year” within a solar cycle.

The natural time interval associated with the motions of the Moon and of the Sun as seen from the Earth is the synodic month, which corresponds to the return of the Moon to the same position with respect to the Sun. The new Moon is traditionally taken as the boundary between two consecutive lunar months. A synodic month comprises 29 days and a fraction of a day that is very close to $^1/_2$. Hence, a synodic month of about $29^1/_2$ days covers an interval of 30 days. The “age of the Moon” is the number of days elapsed since the immediately preceding new Moon. A “schematic lunar month” is the approximation of the synodic month to $29^1/_2$ days, counted from one new Moon to the next and embedded in a calendar year. Such an embedding is usually put into effect by alternating lunar months of 30 or 29 days.[31] A “lunar cycle” is any period after which the sequence of pairings between calendar dates and ages of the Moon repeats itself.

The 19-year lunar cycle comprises 19 calendar years of 365 days, which equal 6935 days; these are organized as a sequence of 228 alternating lunar months of 30 and 29 days (= 6726 days) plus 7 “embolismic” (ἐμβόλιμοι) months of 30 days each (= 210 days) occurring in specific years and resulting from the fact that 12 lunar months of $29^1/_2$ days correspond to only 354 days. The 11 days needed to complete a calendar year of 365 days accumulate (the quantity accumulated at each lunar cycle year is called “epacts”, see sect. **6**) until they exceed 30 days; when this happens, an embolismic lunar month of 30 days is formed, and these days are subtracted from the accumulated epacts. In this case, a calendar year comprises 13 lunations, and the “lunar year” has 13 months. Accordingly, a 19-year lunar cycle comprises 228 + 7 = 235 lunar months of 30 or 29 days. These 235 lunar months comprise 6936 days: the difference of 1 day between the 6935 days counted on the calendar and the 6936 days counted according to the age of the Moon is eliminated by inserting a *saltus lunae*—that is, by increasing the age of the Moon by one day at some point of its cycle: in Byzantine Computi, the *saltus lunae* is normally inserted towards the end of the 16th lunar cycle year.[32] A “lunar cycle year” is a calendar year whose beginning can be shifted with respect to the beginning of the civil (calendar) year. A 19-year cycle consists thus of 19 calendar years, 19 lunar cycle years, and 19 “lunar years” (the latter of variable length, as they can be either 12-month or 13-month sequences); these three 19-“year” periods

[31] The pattern of embedding is a “lunar calendar”, see L. Holford-Strevens, “Paschal Lunar Calendars up to Bede”, *Peritia* 20 (2008), 165-208.

[32] See the list in Grumel, *La Chronologie* (cit. n. 19), 54-55. As a lunar day is eliminated by means of the *saltus lunae*, the epacts at the end of the 16th lunar cycle year increase by 12 units.

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overlap but differ from one another because different meanings of "year" are involved. In lunar computations, leap years are disregarded (see sect. **8**).

In Byzantine Computi, the solar cycle, the lunar cycle and the reference era are synchronized: year 1 of the Byzantine world era is also year 1 of the solar and lunar cycles. For this reason, the reduction rules from world era years to solar and lunar cycle years are straightforward;[33] the algorithms for these very rules are the first given by Rhabdas in sects. **4** and **5**. This reduction is carried out by eliminating whole solar or lunar cycles from the total of world era years.

4

Ὅτε οὖν βούλει τὸν τοῦ ἡλίου κύκλον εἰδέναι, κράτησον τὰ ἀπὸ κτίσεως κόσμου εὑρισκόμενα ἔτη, καὶ ταῦτα μέρισον παρὰ τὸν κη, τουτέστι ἔκβαλον ὁσάκις ἐγχωρεῖ τὸν κη, καὶ τὰ κάτωθεν εὑρεθέντα τούτων ὁ τοῦ ἡλίου κύκλος ἐστίν. ὑφείλομεν οὖν τὰ ,ςων ἐπὶ τῶν κη οὕτως, καὶ λέγομεν· κη σ, ,εχ· κη μ, ,αρκ· κη δ, ρλβ[a]· ἔμειναν λοιπὰ καὶ ιη. καὶ ἔστιν ὁ τοῦ ἡλίου κύκλος ιη.

Ἔτι καὶ διὰ τῆς ἑτέρας συντομωτέρας μεθόδου τὸν τοῦ ἡλίου κύκλον εὑρεῖν, ποίει οὕτως. κράτησον τὰ κάτωθεν, ὡς εἴρηται, τῶν ,ςω ἐτῶν εὑρισκόμενα ἔτη, τουτέστι τὰ ν, καὶ τούτοις πρόσθες καὶ κδ, ἅτινα δηλονότι καὶ ἐναπελείφθησαν ἀπὸ τῶν ,ςω ἀφαιρουμένων παρὰ τῶν κη, καὶ γίνονται τὰ ὅλα οδ· ἐκ τούτων ἄφελε ὁσάκις δύνῃ τὸν κη, καὶ δύνασαι πάντως τοῦτον ἐκβαλεῖν δίς· ἐναπελείφθησαν καὶ ιη, ἴσα καὶ ταῦτα ὄντα τῇ προτέρᾳ μεθόδῳ.

[a] lege ριβ

Algorithms for finding the solar cycle year

Then, whenever you wish to know the cycle of the Sun, keep the years found from the foundation of the world, and divide these by 28, that is, cast 28 aside as many times as possible, and that which is found down from these is the cycle of the Sun. Then, we remove 6850 by 28 as follows, and we say: 28 <times> 200, 5600; 28 <times> 40, 1120; 28 <times> 4, 112; there also remain 18 as a remainder. And the cycle of the Sun is the 18th.

Further, to find the cycle of the Sun by means of the other, more concise, algorithm too, do as follows. Keep, as said, that which is found down from 6800 years, that is, 50, and add 24 to these too, which, as is clear, are also left out from 6800 once they are removed by 28, and they yield 74 as a whole; remove 28 as many times as you can from

[33] Synchronization is not exact since, as seen in sects. **2** and **3**, all these years begin on different dates: therefore, segments of two consecutive solar or lunar cycle years belong to one and the same calendar year. However, Passover, Easter, and most movable feasts of the Christian calendar fall in the "safe" time interval bounded by January 1 and August 31.

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these, and you can always cast this aside twice; 18 are also left out, which are also equal to <those resulting with> the previous algorithm.

Par. The algorithm for finding the solar cycle year *s* of an assigned year *y* in the Byzantine era is:

$(y) \rightarrow y \bmod 28 = s$.

A computation is carried out for current year AM 6850 [= 1341/2], and it yields $y = 6850 \rightarrow s = 18$.

A more concise algorithm for finding the solar cycle year is:

$(y) \rightarrow y - 6800 \rightarrow (y - 6800) + 24 \rightarrow [(y - 6800) + 24] \bmod 28 = s$.

This algorithm relies on the fact that $24 \equiv 6800 \pmod{28}$. A computation is carried out for current year AM 6850 [= 1341/2], and it yields $y = 6850 \rightarrow s = 18$.

5

Ὁ δὲ τῆς σελήνης κύκλος ἄρχεται ἀπὸ τῆς πρώτης τοῦ Ἰαννουαρίου μηνός, καὶ ἀνέρχεται εἰς χρόνους ιθ, καὶ πάλιν ἄρχεται πρῶτος. ὅτε οὖν βούλει τὸν ταύτης κύκλον εὑρεῖν, κράτησον τὰ ἀπὸ κτίσεως κόσμου ἔτη, ἅ εἰσι κατὰ τὴν σήμερον ,ςων, καὶ ἀνάλυσον ταῦτα παρὰ τὸν ιθ, καὶ τὰ εὑρισκόμενα κάτωθεν τῶν ιθ ὁ τῆς σελήνης κύκλος ἐστίν. ἄφελε οὖν οὕτως αὐτά, καὶ εἰπέ· ιθ τ, ,εψ· ιθ ξ, ,αρμ· λοιπὰ οὖν ἔμειναν καὶ ι. καὶ ἔνι ὁ τῆς σελήνης κύκλος ι.

Ἔτι καὶ μετὰ τῆς συντόμου μεθόδου τὸν ταύτης κύκλον εὑρεῖν, ποίει οὕτως. κράτησον τὰ ὀλίγα τῶν ἐτῶν, ὡς ἀνωτέρω σοι εἴρηται, τουτέστι τὰ ν, καὶ πρόσθες τούτοις καὶ ιζ – ταῦτα γὰρ καὶ μόνα καταλιμπάνονται ἀπὸ τῶν ,ςω μεριζομένων παρὰ τῶν ιθ, ἃ δὴ καὶ ὥσπέρ τινα ῥίζαν καὶ θεμέλιον ἔχομεν –· γίνονται ὁμοῦ ξζ· ταῦτα ἄφελε παρὰ τὸν ιθ, καὶ εἰπέ· τρὶς ιθ, νζ· ἐναπελείφθησαν καὶ ι, ἴσα καὶ ταῦτα τῇ προτέρᾳ μεθόδῳ.

The lunar cycle, and algorithms for finding the lunar cycle year

The cycle of the Moon begins on the first of the month of January, and reaches to 19 years, and begins again first. Then, whenever you wish to know the cycle of this, keep the years from the foundation of the world, which are 6850 to-day, and resolve these out into 19, and that which is found down from 19 is the cycle of the Moon. Then, remove them as follows, and say: 19 <times> 300, 5700; 19 <times> 60, 1140; then, there also remain 10 as a remainder. And the cycle of the Moon is the 10^{th}.

Further, to find the cycle of this with the concise algorithm too, do as follows. Keep, as said above to you, the small parts of the years, that is, 50, and add 17 to these too—for these and only these are left out from 6800 once they are divided by 19, which we also

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regard quite as a root and base of sorts—together they yield 67; remove these by 19, and say: thrice 19, 57; 10 are also left out, which are also equal to <those resulting with> the previous algorithm.

Par. The lunar cycle begins on January 1 and lasts 19 years. The algorithm for finding the lunar cycle year m of an assigned year y in the Byzantine era is:

$(y) \rightarrow y \bmod 19 = m$.

A computation is carried out for current year AM 6850 [= 1341/2], and it yields $y = 6850 \rightarrow m = 10$.

A concise algorithm for finding the lunar cycle year (the quantity $y - 6800$ is called "the small parts of the years" [τὰ ὀλίγα τῶν ἐτῶν]) is:

$(y) \rightarrow y - 6800 \rightarrow (y - 6800) + 17 \rightarrow [(y - 6800) + 17] \bmod 19 = m$.

This algorithm relies on the fact that $17 \equiv 6800 \pmod{19}$. A computation is carried out for current year AM 6850 [= 1341/2], and it yields $y = 6850 \rightarrow m = 10$.

6

Ὁ δὲ ταύτης θεμέλιος καταλαμβάνεται οὕτως. ἑνδεκαπλασίασον τὸν τῆς σελήνης κύκλον οἷος ἐστί, καὶ τῷ γινομένῳ ἀριθμῷ ἀπὸ τοῦ πολλαπλασιασμοῦ πρόσθες καὶ γ, ἃς φασὶν ἀφωτίστους διὰ τοὺς φωστῆρας τῇ τετάρτῃ ἡμέρᾳ γενέσθαι, καὶ ἐξ αὐτοῦ ἄφελε ὅσας εὕρῃς τριακοντάδας, καὶ τὰ καταλειφθέντα κάτωθεν τῶν λ ὁ τῆς σελήνης ὑπάρχει θεμέλιος.

|65r Ὑποδείγματος δὲ χάριν εὑρέθη κατὰ τὸ παρὸν ,ςων ἔτος ὁ τῆς σελήνης κύκλος δέκατος, καὶ λέγομεν ὅτι ἑνδακάκις τὰ ι, ρι· τούτοις προστίθεμεν καὶ γ· καὶ γίνονται ὁμοῦ ριγ, ἐξ ὧν ἀφαιροῦμεν τὸν λ τρίς· καὶ καταλιμπάνονται κγ. λέγομεν οὖν εἶναι καὶ τὸν ἐνεστῶτα νῦν τῆς σελήνης θεμέλιον εἰκοστότριτον.

An algorithm for finding the base of the Moon

The base of this [*scil*. the Moon] is taken as follows. Undecuple the cycle of the Moon what<ever> it is, and add 3, which they call dark <days> because the luminaries have come to be in the fourth day, to the number resulting from the multiplication too, and remove how many thirties you find from it, and that which is left down from 30 turns out to be the base of the Moon.

For example, in the present year 6850, the cycle of the Moon has been found to be the tenth, and we say that eleven times 10, 110; we also add 3 to these; and together they yield 113, from which we remove 30 thrice; and 23 are left out. Then, we say that the now-present base of the Moon is also the twenty-third.

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Par. The algorithm for finding the base (θεμέλιος) of the Moon b_m at lunar cycle year m is:

$(m) \rightarrow 11m \rightarrow 11m + 3 \rightarrow (11m + 3) \bmod 30 = b_m$.

The additive parameter 3 (the so-called "dark <days>" [ἀφώτιστοι]) comes from the fact that the two luminaries came to be on the fourth day of Creation. A computation is carried out for current year AM 6850 [= 1341/2], and it yields $m = 10 \rightarrow b_m = 23$.

Comm. Each lunar year (= 354 days) is 11 days shorter than a 365-day calendar year; this difference accumulates. The lunar "epacts" (ἐπακταί, litt. the "<days> brought upon") are the difference that is accumulated at an assigned lunar cycle year within a 19-year lunar cycle (see sect. **8**).[34] Whenever this cumulative difference is greater than 30 days, these 30 days make an "embolismic" month and are thereby subtracted from the epacts. In a lunar cycle that is synchronized with January 1, the epacts coincide with the age of the Moon on December 31. For this reason, a "base" of the Moon b_m was introduced such that b_m = epacts + 1, which is but the age of the Moon on January 1. A base adapted to the features of some specific algorithms and defined by b_m = epacts + 3, was also introduced: this is Rhabdas' base.[35] As I shall explain (see sect. **7**), he should, in fact, have used the "epacts + 1" base. Since, in the Byzantine 19-year cycle, the first lunar cycle year has 11 epacts, Rhabdas' bases, keyed to lunar cycle years, are as in the following table (note the absence of the *saltus lunae* between cycles 16 and 17, and see sect. **10**):

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
b_m	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29	10	21	2

7

Μεθὸ δὲ εὕρῃς τὸν τῆς σελήνης θεμέλιον, καὶ θέλεις εὑρεῖν καὶ τὸ πόσων ἡμερῶν ἐστὶν ἡ σελήνη ἀπὸ συνόδου ἢ πανσελήνου, τουτέστιν ἀπὸ νεομηνίας καὶ ἀποχύσεως, κράτησον τὸν εὑρεθέντα θεμέλιον, καὶ ἄρξου ἀπὸ τοῦ Ἰαννουαρίου μηνός, καὶ ἐφεξῆς τοὺς παρελθόντας μῆνας ὅλους προστιθέναι τῷ τοιούτῳ θεμελίῳ πάσας τὰς τῶν μηνῶν ἡμέρας μέχρι καὶ τῆς ποστῆς τοῦ μηνὸς ἐν ᾗ τὴν ζήτησιν ποιεῖς, καὶ συναγαγὼν πάσας καὶ ἀθροίσας εἰς μίαν ποσότητα ἄφελε ἐξ αὐτῶν ὅσους εὕρῃς μῆνας σεληνιακούς – ἡ δὲ τοῦ

[34] For lunar epacts in Byzantine Computi, see the early and clear expositions by George Presbyter (dated 638/9), sect. 2, in F. Diekamp, "Der Mönch und Presbyter Georgios, ein unbekannter Schriftsteller des 7. Jahrhunderts", *Byzantinische Zeitschrift* 9 (1900), 14-51, at 25, and by Maximus the Confessor, *Brevis Enarratio Christiani Paschatis* (dated 640/1), sect. I.7, in *PG* XIX, 1217-1279, at 1223; see also such an early Computus as *Anonymus* 892, sect. 14.

[35] For this "base", see, for instance, *Anonymus* 1247, sect. 20, in Schissel, "Chronologischer" (cit. n. 30); the last section of *Anonymus* 1256; Matthew Blastares, in Rhalles, Potles, Σύνταγμα (cit. n. 30), 414-415 and 416-417; Isaac Argyros, sect. 7, in *PG* XIX, 1279-1316, at 1293; *Anonymus* 1377, sect. 5, in *PG* XIX, 1316-1329, at 1321; *Anonymus* 1379 or Pseudo-Andreas, in *PG* XIX, 1329-1334, at 1334. See also the list of epacts and bases in Grumel, *La Chronologie* (cit. n. 19), 54-55.

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σεληνιακοῦ μηνὸς ποσότης εἰσὶν ἡμέραι κθ ∸ καὶ λεπτὰ γ· οὕτω γὰρ δοκεῖ παραδοῦναι ἡμῖν ὁ μαθηματικὸς Πτολεμαῖος – καὶ τὰς κάτωθεν τῶν κθ ∸ εὑρισκομένας νόει εἶναι τὰς ἀκριβεῖς τῆς σελήνης ἡμέρας.

τινὲς δὲ διὰ τὸ ἀσύγχυτόν τε καὶ εὔληπτον τριακοντάδας ἀφαιροῦσι, καὶ ὕστερον τῷ καταλειφθέντι ἀριθμῷ προστιθέασιν ὑπὲρ μιᾶς ἑκάστης τριακοντάδος ἡμέρας τὸ ἥμισυ. ἄλλοι δὲ ἀφαιροῦσι ἑξηκοντάδας, εἴπερ εἰς μείζονα ὄγκον ὁ ἀριθμὸς προχωρεῖ, καὶ ὑπὲρ μιᾶς ἑκάστης ἑξηκοντάδος προστίθουσι τῷ καταλειφθέντι ἀριθμῷ ἡμέραν μίαν διὰ τὸ εἰς νθ ἡμέρας ποσοῦσθαι τοὺς δύο σεληνιακοὺς μῆνας. καὶ αὕτη μέν ἐστιν ἡ καθολικὴ καὶ ἀκριβὴς μέθοδος.

The age of the Moon on an assigned date

After you have found the base of the Moon, and you also want to find the age of the Moon from conjunction or from full Moon, that is, from new Moon and blooming, keep the found base, and begin from the month of January, and successively add all of the bygone months to such a base, <namely,> all the days of the months up to and including the date of the month in which you carry out your search, and gathering all of them and putting them together into a single quantity remove how many lunar months you find from them—the quantity of a lunar month are 29½ days and 3 minutes, for the astronomer Ptolemy appears to hand so-and-so down to us—and consider that those which are found down from 29½ are the exact age of the Moon.

Par. The algorithm for finding the age of the Moon $a(x_X)$ on day x in month X in a given lunar cycle year m, counting from the day of the new Moon (σύνοδος, litt. "conjunction", νεομηνία) or of the full Moon (πανσέληνος, ἀπόχυσις, litt. "blooming"), with n_k = number of days in month k, is the following:

$$(b_m,x,X) \rightarrow b_m + \sum_{k=J}^{X-1} n_k + x \rightarrow \left(b_m + \sum_{k=J}^{X-1} n_k + x\right) \bmod 29\tfrac{1}{2} = a(x_X).$$

The length of the lunar month is of 29½ days and 3 minutes according to Ptolemy, and this is exactly what should be subtracted in the modulo reduction.

Comm. The age of the Moon on day x in month X is found by adding its age on a date taken as epoch to the elapsed days counted from that date, and by then removing whole lunar months. Since Rhabdas begins counting from January 1, his use of the base b_m = epacts + 3 is incorrect. The counting of the days elapsed from epoch is carried out by grouping the days of the elapsed months (addendum $\sum_{k=J}^{X-1} n_k$), to which the days x counted in the last month X must be added. The final reduction modulo 29½ removes whole lunar months.[36] Rhabdas' statement about the

[36] Computi that present algorithms in which the age of the Moon is calculated by reducing modulo 29½ include those by Maximus the Confessor, sect. I.28 and the eighth algorithm compiled in sect. III.8, in *PG* XIX, 1245 and 1269; Psellos (dated 1091/2), sect. I.15, in G. Redl, "La

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3 minutes is unclear, for the mean synodic month according to *Almagest* IV.2 exceeds $29\frac{1}{2}$ days by a little less than 2 minutes (for it is 29;31,50,8,9,24 days) and a λεπτόν according to the calculations given in the next section (but unlike those found in sect. **9**) is $\frac{1}{60}$ a day.

Alternative algorithms for the age of the Moon at an assigned date

However, both in order to avoid confusion and for an easy apprehension, some remove thirties, and afterwards they add half a day for each single thirty to the number left out. Others remove sixties, whenever the number proceeds to a greater amount, and they add a day for each single sixty to the number left out because two lunar months amount to 59 days. And this is the general and exact algorithm.

Par. An alternative, clear and easy-to-understand algorithm is:

$(b_m, x, X) \to b_m + \sum_{k=J}^{X-1} n_k + x \to (b_m + \sum_{k=J}^{X-1} n_k + x) \bmod 30 + \frac{1}{2}[\![(b_m + \sum_{k=J}^{X-1} n_k + x)/30]\!] = a_m(x_X)$.

The last addendum reintegrates $\frac{1}{2}$ a day, of which one falls short when reducing modulo 30 instead of modulo $29\frac{1}{2}$.

An alternative algorithm to be used when the addition yields a large sum is:

$(b_m, x, X) \to b_m + \sum_{k=J}^{X-1} n_k + x \to (b_m + \sum_{k=J}^{X-1} n_k + x) \bmod 60 + [\![(b_m + \sum_{k=J}^{X-1} n_k + x)/60]\!] = a_m(x_X)$.

The last addendum reintegrates 1 day, of which one falls short when reducing modulo 60, since 2 lunar months last 59 days.

Comm. These algorithms, whose rationale is explained by Rhabdas in detail, are also given in *Anonymus* 1183, sect. 9. Lunar age algorithms usually simplify matters and reduce modulo 30 without reintegrating a day or a fraction of it.[37] Algorithms that reduce modulo 60 can also be found, especially in connection with the approach advocated by the so-called πενταπλοῦντες καὶ ἑξαπλοῦντες.[38]

chronologie appliquée de Michel Psellos", *Byzantion* 4 (1927-28), 197-236; 5 (1929-30), 229-286: II, 237.1-11; *Anonymus* 1183, sects. 9 and 10, edited in F. Acerbi, "Struttura e concezione del vademecum computazionale Par. gr. 1670", *Segno e Testo* 19 (2021), 167-255; *Anonymus* 1377, sect. 6, in *PG* XIX, 1323.

[37] See the compilation in Maximus the Confessor, sect. III.8, in *PG* XIX, 1268-1269; Psellos, sect. I.8, in Redl, "La chronologie" (cit. n. 36), I, 221-223; *Anonymus* 1092A, sect. 10, in Karnthaler, "Die chronologischen Abhandlungen" (cit. n. 30), 7.109-113; *Anonymus* 1247, sects. 21 and 25, in Schissel, "Chronologischer" (cit. n. 30), 109 and 110; Isaac Argyros, in *PG* XIX, 1294-1296.

[38] For the latter, see Maximus the Confessor, sects. I.11-12, 16, in *PG* XIX, 1228-1229, 1233, and the entire chapter II, in *PG* XIX, 1252-1264; *Anonymus* 1079, sect. 5, in A. Mentz, *Beiträge zur Osterfestberechnung bei den Byzantinern*, Diss. Königsberg 1906, 76-100, at 80-84, and also the discussion at 51-66; *Anonymus* 892, sect. 8; *Anonymus* 1092A, sect. 3 (this Computus is a copy of *Anonymus* 892). The latter defines the algorithm as χαρτουλαρικός "archive-keeper-style":

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Ἄλλοι δὲ διὰ τὸ σύντομον ὁμοῦ τε καὶ ῥᾴδιον ἀφαιροῦντες ἀφ᾽ ἑκάστου μηνὸς ἡλιακοῦ μῆνα σεληνιακὸν κατέχουσι τὰς περιττευούσας ἡμέρας ἐξ ἑκάστου μηνός, ἃς δὴ καὶ ἐπακτὰς ὀνομάζουσι. περιττεύουσιν οὖν ἀπὸ μὲν τῶν μηνῶν τοῦ ἡλίου τῶν ἐχόντων ἡμέρας λα, ἡμέρα μία καὶ ἡμίσεια, ἀπὸ δὲ τῶν ἐχόντων λ, ἡμίσεια καὶ μόνη· συνάγονται οὖν ἐκ τῶν ιβ μηνῶν ἡμέραι ια, ἃς καὶ τῷ ἐνεστῶτι θεμελίῳ κατέτος προστιθέντες τὸν ἐπιόντα εὑρίσκομεν θεμέλιον, εἰ μὴ ὑπερβῇ τὸν λ· εἰ δὲ ὑπερβῇ τὸν λ, ἀφαιροῦντες αὐτὸν κρατῶμεν τὰ καταλειφθέντα.

Ἀπορέσειε δ᾽ ἄν τις δικαίως, ἀκούσας ὅπως ιβ ὄντων τῶν τοῦ ἡλίου μηνῶν καὶ τῶν ζ ἐχόντων ἀνὰ λα ἡμέραν· πῶς οὐκ εἰσὶ αἱ ἐπακταὶ ιγ ἀλλὰ ια; καὶ ἀπολογούμεθα πρὸς τοῦτο ὅτι τὴν περιττεύουσαν μίαν καὶ ἡμίσειαν ἡμέραν τοῦ Ἰαννουαρίου μηνὸς ἀπὸ τοῦ σεληνιακοῦ μηνός, λαμβάνει ταύτην ὁ Φευρουάριος διὰ τὸ ἔχειν τοῦτον ἡμέρας κη καὶ μὴ ἐξισοῦσθαι μηνὶ σεληνιακῷ, κἀντεῦθεν τὰς ια γεννᾶσθαι ἀπὸ μόνων τῶν δέκα μηνῶν. ὅταν δὲ τύχῃ ὁ χρόνος γενέσθαι βίσεξτος, τότε οὐκ ἕνδεκα γίνονται αἱ τῶν μηνῶν ἐπακταὶ ἀλλὰ ιβ διὰ τὸ ἔχειν τότε τὸν Φευρουάριον ἡμέρας κθ, ἥντινα δηλονότι ἡμέραν δέχεται ἀπὸ τῆς τοῦ βισέξτου προσθήκης· δανείζεται γὰρ τότε ἀπὸ τοῦ Ἰαννουαρίου ὁ Φευρουάριος ἡμίσειαν καὶ μόνην ἡμέραν, καὶ ἐξ ἀνάγκης ἀπομένει ἡ μία τοῦ Ἰαννουαρίου ἡμέρα. συνάγονται οὖν |65v αἱ ἐπακταὶ ἐκ τῶν ι μηνῶν, [ὅτε μὲν οὐ][a] τύχῃ ὁ χρόνος βίσεξτος, οὕτως. Μάρτιος α 𐅵 διότι ἔχει ἡμέρας λα, Ἀπρίλλιος ἡμίσειαν διότι ἔχει λ, καὶ καθεξῆς Μάϊος α 𐅵, Ἰούνιος ἡμίσειαν, Ἰούλιος α 𐅵, Ἀύγουστος α 𐅵, Σεπτέμβριος 𐅵, Ὀκτώβριος α 𐅵, Νοέμβριος ἡμίσειαν, Δεκέμβριος α 𐅵· ὁμοῦ ια. ὅτε δὲ τύχῃ ὡς εἴρηται γενέσθαι τὸν χρόνον βίσεξτον, τότε περιττεύει καὶ ἡ μία τοῦ Ἰαννουαρίου, καὶ γίνονται ιβ· τὴν γὰρ ἡμίσειαν τούτου δέχεται ὁ Φευρουάριος, ὡς δεδήλωται. καὶ ταῦτα σοι διεξῆλθον ἵνα ἔχεις ἀκριβῶς εἰδέναι ταῦτα καὶ καταλόγον.

Ὑποδείγματος οὖν χάριν ἔστω ζητεῖν ἡμᾶς εὑρεῖν κατὰ τὴν ἐννάτην τοῦ Μαρτίου μηνὸς πόσων ἡμερῶν ἐστὶν ἡ σελήνη. καὶ λέγομεν κατὰ τὴν καθόλου μέθοδον ὅτι κγ θεμέλιος, Ἰαννουαρίου λα, Φευρουαρίου κη καὶ Μαρτίου θ· γίνονται ὁμοῦ ϙα· ἐκ τούτων ἀφαιρῶ [[ἡμέρας]] τριακοντάδας τρεῖς (ἢ[b] μίαν καὶ ἡμίσειαν ἑξηκοντάδα· ταυτὸν γάρ ἐστιν), καὶ ἐναπελείφθη μοι μία· ταύτῃ προστίθημι καὶ τὰς ἀπὸ τῶν τριῶν τριακοντάδων περιττευούσας τρεῖς ἡμισείας ἡμέρας, αἳ καὶ ποιοῦσιν ἡμέραν μίαν καὶ ἡμίσειαν· καὶ γίνονται μετὰ τῆς προτέρας μιᾶς ἡμέραι β 𐅵. καὶ λέγω εἶναι τὴν σελήνην κατὰ τὴν θην ὅλην τοῦ Μαρτίου μηνὸς ἡμέραν ἡμερῶν β τρίτον ἔγγιστα, ἐπειδὴ ἀφαιρῶ καὶ θ λεπτὰ ὑπὲρ τῶν γ φεγγηαρίων[c].

Ὡσαύτως καὶ διὰ τῆς ἑτέρας μεθόδου καὶ ταύτην ὑποδειγματιστέον, καὶ ἔστω ζητεῖν ἡμᾶς εὑρεῖν κατὰ τὴν ιδ τοῦ ἐρχομένου Σεπτεμβρίου τῆς ιαης ἰνδίκτου πόσων

Karnthaler, "Die chronologischen Abhandlungen" (cit. n. 30), 5.28. *Anonymus* 1183, sect. 9, sets out a modulo 60 algorithm similar to Rhabdas'.

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ἡμερῶν μέλλει εὑρεθῆναι ἡ σελήνη, καὶ λέγομεν οὕτως. κγ θεμέλιος· Ἰαννουάριος καὶ Φευρουάριος ἐξισοῦνται καὶ οὐδὲν ἔχουσι· καὶ ἄρχομαι ἀπὸ τοῦ Μαρτίου, καὶ λέγω· Μάρτιος α ∸, Ἀπρίλλιος ∸, Μάϊος α ∸, Ἰούνιος ∸, Ἰούλιος α ∸, Ἀύγουστος α ∸ καὶ Σεπτέμβριος ιδ· γίνονται ὁμοῦ μδ· ἐκ τούτων ἀφαιρῶ κθ ∸ καὶ δ ∸ λεπτά· καὶ ἐναπελείφθησαν ιδ καὶ κε ∸ λεπτά, ὡς εἶναι ταύτην ἔγγιστα πανσέληνον. εὑρίσκεται οὖν καὶ διὰ ταύτης τῆς μεθόδου κατὰ τὴν δοθεῖσαν ιδ τοῦ Σεπτεμβρίου ἡ σελήνη ἡμερῶν ιδ γ^{ον} ιβ^{ον} καὶ ρκ^{ον}, τουτέστιν ἡμερῶν ιδ καὶ λεπτῶν κε ∸. τοσαῦτα σοι καὶ περὶ τῶν ἡμερῶν τῆς σελήνης. ἑπόμενον λοιπόν ἐστιν εἰπεῖν καὶ περὶ τῆς φαύσεως τῶν ἡμερῶν[d] αὐτῆς, ἤγουν πόσας ὥρας λάμπει καὶ δᾳδουχεῖ καθ' ἑκάστην νύκτα.

[a] μινῶνο [sp. 2 litt.] ἡ μεν^{ου} scr. m.2 [b] scripsi : legi nequit fort. scripsit εἰ [c] lege φεγγαρίων [d] lege ὡρῶν cf. tit.

The epacts of the Moon

Others, for the sake of both conciseness and easiness together, removing a lunar month from each solar month hold the days that remain over from each month, which they really also call "epacts". Then, one day and a half remain over from the months of the Sun that have 31 days, and one half only from those that have 30; then, 11 days are gathered from the 12 months, adding which year by year to the present base too we find the subsequent base, if this does not overstep 30; if indeed it oversteps 30, let us keep what is left out after removing it.

Someone might legitimately raise the following objection, when hearing that the months of the Sun are 12 and 7 of them have 31 days each: how on earth the epacts are not 13 but 11? And we reply to this that February takes the one day and a half that remains over from the lunar month in the month of January because this [*scil*. February] has 28 days and it is not equal to a lunar month, whence the 11 <epacts> arise from ten months only. Whenever the year happens to be a leap year, then the epacts of the months do not come to be eleven but 12 because February has then 29 days, which <extra> day, as is clear, it receives from the addition of the leap day, for February then borrows only a half a day from January, and by necessity one day of January remains. Then, the epacts are gathered from 10 months, whenever the year does not happen to be a leap year, as follows. March $1\frac{1}{2}$ because it has 31 days, April one half because it has 30, and in succession May $1\frac{1}{2}$, June one half, July $1\frac{1}{2}$, August $1\frac{1}{2}$, September $\frac{1}{2}$, October $1\frac{1}{2}$, November one half, December $1\frac{1}{2}$; together 11. Whenever, as said, the year does happen to be a leap year, then one day of January also remains over, and <the epacts> become 12, for February receives half of this, as clarified. And go carefully through these <arguments>, in order for you to get these things exactly and on a rational basis.

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Then, for example, let there be a search for us to find the age of the Moon on the ninth of the month of March. And according to the general algorithm we say that base 23, January 31, February 28, and March 9; together they yield 91; I remove from these three thirties (or one sixty and a half, for it is the same), and one <day> is left out for me; I also add to it the three halves of a day that remain over from the three thirties, which also make one day and a half; and they yield, with the previous single <day>, 2½ days. And I say that, on the whole 9th day of the month of March, the age of the Moon is very nearly 2 <days> and a third, since I also remove 9 minutes on behalf of the 3 lunar months.

Likewise, one must also exemplify this by means of the other algorithm too, and let there be a search for us to find what age of the Moon should be found on the 14th of the forthcoming September of the 11th indiction, and we say as follows. Base 23; January and February are equal <to two lunar months> and do not contribute anything; and I begin from March, and I say: March 1½, April ½, May 1½, June ½, July 1½, August 1½, and September 14; together they yield 44; I remove 29½ and 4½ minutes from these; and 14 and 25½ minutes are left out, so that this is very nearly a full Moon. Then, on the given September 14, the age of the Moon is found, by means of this algorithm too, 14⅓ 1/12 1/120 <days>, that is, 14 days and 25½ minutes. So much for you about the age of the Moon too. Consequently, it is next to be also said about the hours of its visibility, namely, how many hours it shines and carries its torch alight on every night.

Par. For the sake of conciseness and easiness, some people use the epacts of the Moon, which are the number of days of excess of a calendar month over a lunar month of 29½ days. This excess is 1½ day for the months that have 31 days, ½ a day for the months that have 30 days. In this way, 11 epacts cumulate every year. What follows is the algorithm for computing the base of the Moon of lunar cycle year $m + 1$ once the base of year m is known:

$(b_m) \rightarrow b_m + 11 \rightarrow (b_m + 11) \bmod 30 = b_{m+1}$.

A difficulty (ἀπορέσειε δ' ἄν τις) and its solution: If the months are 12 and 7 of them have 31 days, why are the yearly epacts 11 and not 13? Because February has 28 days, and the 1½ day needed to complete a whole lunar month amounts exactly to the same excess in January, so that the yearly epacts result from adding the excess of the remaining 10 months. In leap years, the epacts are 12 and not 11 because February has 29 days. A list of the lunar epacts of each month from March to December is given; together they yield 11.

A computation of the age of the Moon is carried out by means of the second and of the third algorithm given in sect. **7**, on March 9 of the current year, and it yields base of the Moon $b_m = 23 \rightarrow$ age of the Moon $a_m(x_X) = 2½$ days, from which Rhabdas subtracts 9 minutes because

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of the intervening 3 lunar months. It yields very nearly $2\frac{1}{3}$ days (for $\frac{9}{60}$ a day are very nearly $\frac{1}{6}$ a day).

A second computation of the age of the Moon is carried out by means of first algorithm given in sect. **7**, on September 14 of the following calendar year, and it yields $i = 11$, $b_m = 23$[39] $\rightarrow a_m(x_X) =$ (by subtracting $29\frac{1}{2}$ days and $4\frac{1}{2}$ minutes from the sum of base, months, and days, which amounts to 44 days) = 14 days and $25\frac{1}{2}$ minutes = $14\frac{1}{3}\frac{1}{12}\frac{1}{120}$ days.

Comm. The last equality is valid because $\frac{1}{3}\frac{1}{12}\frac{1}{120} = \frac{51}{120}$ a day, that is, $\frac{51}{2} = 25\frac{1}{2}$ minutes. Rhabdas' statement that in leap years the epacts are 12 and not 11 because February has 29 days is erroneous: no elements pertaining to lunar computations take leap years into account (in fact, they simply cannot).[40]

9

Ἰστέον οὖν ὅτι ἀπὸ συνόδου μέχρι τῆς πανσελήνου καθέκαστον νυχθήμερον τέσσαρσι λεπτοῖς τὸ ταύτης αὔξεται φῶς, ἀπὸ δὲ πανσελήνου πάλιν μέχρι συνόδου τέσσαρσι μειοῦται λεπτοῖς, ἅτινα λεπτὰ εἰσὶ τέσσαρα πέμπτα τῆς ὥρας, εἴτουν μέρος ὥρας ∽ ιον καὶ λον, ὡς ἐπιβάλλειν τῇ ὥρᾳ λεπτὰ ε. ὁπόταν οὖν ἐθέλεις γνωρίσαι καὶ μαθεῖν πόσας ὥρας φαίνῃ καθ᾽ ἑκάστην ἡ σελήνη νύκτα, τετραπλασίαζε τὰς ἡμέρας αὐτῆς, καὶ τὸν γινόμενον ἀριθμὸν μέριζε παρὰ τὸν ε, καὶ ὅσας ἐκβάλεις πεντάδας, τοσαύτας ὥρας ἀποφαίνου μαρμαίρειν τὴν μῆνιν.

Οἷον εὑρέθη τυχὸν ἡ σελήνη ἡμερῶν ια, καὶ ζητῶμεν μαθεῖν πόσας ἐν ἐκείνῃ τῇ νυκτὶ μέλλει δᾳδουχεῖν ἡ σελήνη, καὶ λέγομεν κατὰ τὴν ῥηθεῖσαν μέθοδον ὅτι τετράκις τὰ ια, μδ· ταῦτα μεριστέον παρὰ τὰ ε, καὶ λέγω· πεντάκις τὰ θ, με παρὰ πέμπτον ἕν. λέγω οὖν ὅτι μέλλει δᾳδουχεῖν κατ᾽ ἐκείνην τὴν νύκτα ὥρας θ παρὰ λεπτὸν α, ὅ ἐστι εον, ἢ ὥρας η καὶ λεπτὰ δ, |66r τουτέστιν ὥρας μιᾶς ∽ ιον καὶ λον.

ἀλλ᾽ αὕτη μὲν ἡ μέθοδος ἐν τῷ καιρῷ τῆς ἰσημερίας μόνον ἐπαληθεύσει – ἴσως δὲ καὶ καιρικῶν τῶν ὡρῶν νοουμένων κατὰ τὸν ἅπαντα χρόνον, εἴτουν καὶ ιβ ὡρῶν νοεῖσθαι τήν τε νύκτα καὶ τὴν ἡμέραν καὶ τὰς μὲν νοεῖν μείζονας τὰς δὲ ἐλάσσονας, ἰσημερινῶν δὲ λογιζομένων ἐν παντὶ καιρῷ τῶν ὡρῶν, τουτέστιν ἐξίσου διαστήματος εἶναι καὶ τὰς νυκτερινὰς ὥρας καὶ τὰς ἡμερινάς, ὡς φέρε εἰπεῖν τὴν μὲν μείζονα ἡμέραν ὡρῶν εἶναι ιε τὴν δὲ ἐλάσσονα νύκτα ὡρῶν θ, καὶ τὸ ἀνάπαλιν τὴν μὲν μεγίστην [[ἡμέραν]] \νύκταν/ ὡρῶν εἶναι ιε τὴν δὲ ἐλαχίστην ἡμέραν ὡρῶν θ – οὐκ ἀληθεύσει πάντως αὕτη ἡ μέθοδος, ἀλλ᾽ ἑτέρας πάντως δεηθησόμεθα. ῥηθήσεται δὲ καὶ αὕτη οἴκοθεν σὺν θεῷ, καὶ οὐ δείσομεν τὸν ἐλέγξοντα.

[39] In September, the calendar year (and hence the indiction) changes, but the lunar cycle year remains the same, which means that the same "base" must be used.

[40] This aspect is frequently overlooked in analyses of the technical basis of Computi. This point is discussed in Holford-Strevens, "Paschal Lunar Calendars" (cit. n. 31).

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Ἔχει δὲ οὕτως. πολλαπλασίαζε ἀεὶ τὰς εὑρισκομένας τῆς σελήνης ἡμέρας ἐπὶ τὰς ὥρας τῆς νυκτὸς ἐκείνης καθ᾽ ὃν καιρὸν ἡ ζήτησις γίνεται, καὶ τὸν συναχθέντα ἀριθμὸν μέριζε παρὰ τοὺς χρόνους τῆς ἰσημερινῆς ὥρας, οἵτινές εἰσι ιε, καὶ καθ᾽ ἑκάστην πεντεκαιδεκάδα νόει ὥραν μίαν.

Ὑποδείγματος χάριν ἐγένετο ἡ ζήτησις εὑρεῖν ἡμᾶς κατὰ τὸν Ἰαννουάριον μῆνα (καθ᾽ ὃν καιρὸν ἡ νὺξ ἔχει ὥρας ιδ) ἡμερῶν δώδεκα τῆς σελήνης οὔσης πόσας ὥρας μέλλει φαίνειν. καὶ λέγω κατὰ τὴν ῥηθεῖσαν μέθοδον· δωδεκάκις τὰ ιδ, ρξη· ταῦτα παρὰ τὸν ιε μεριζόμενα ποιοῦσιν ὥρας ια καὶ ε^{ον}. καὶ λέγω λάμψαι τὴν σελήνην κατ᾽ ἐκείνην τὴν νύκτα ὥρας ια καὶ ε^{ον}· μετὰ δὲ τῆς προρρηθείσης ἄλλης μεθόδου εὑρίσκονται ὧραι θ 𐅷 καὶ δέκατον.

Ἔτι ὑποδειγματιστέον καὶ καθ᾽ ὃν καιρὸν ἡ νὺξ ὥρας ἔχει θ, τουτέστι κατὰ τὸν Ἰούνιον μῆνα, τῆς σελήνης ἡμερῶν εὑρεθείσης ιδ πόσας ὥρας μέλλει φαύσαι. καὶ λέγω καὶ αὖθις κατὰ τὴν δοθεῖσαν μέθοδον· πολυπλασιάζω τὰς ιδ ἡμέρας τῆς σελήνης ἐπὶ τὰς θ ὥρας τῆς νυκτός· καὶ γίνονται ρκς· ἐνεάκις γὰρ τὰ ιδ, ρκς ποιοῦσι· ἐκ τούτων οὖν δύναμαι τὸν ιε ἀφελεῖν ὀκτάκις· μένουσι καὶ ς, ἅπέρ εἰσι μέρος τῶν ιε πέμπτα δύο, τουτέστιν ὥρας μέρος τρίτον καὶ ιε^{ον}. καὶ ἀποφαίνομαι λάμψαι κατ᾽ ἐκείνην τὴν νύκτα τὴν σελήνην ὥρας ἰσημερινὰς η γ^{ον} καὶ ιε^{ον}. τοσαῦτα σοι καὶ περὶ τῶν τῆς σελήνης ἐφόδων.

Duration of visibility of the waxing and waning Moon at an assigned age of the Moon

Then, one must know that, from conjunction up to full Moon, its light increases by four minutes per each nychthemeron, and again from full Moon up to conjunction, it decreases by four minutes, which minutes are four fifths an hour, viz. the $^2/_3\,^1/_{10}\,^1/_{30}$ part of an hour, in such a way that five minutes cast upon an hour. Then, whenever you want to recognize and to learn how many hours the Moon shines each night, quadruple its age, and divide the resulting number by 5, and how many pentads you cast aside, so many hours declare the Moon shines.

For instance, the age of the Moon happened to be found of 11 days, and let us seek to learn how many <hours> the Moon should shine on that night, and according to the expounded algorithm we say that four times 11, 44; one must divide these by 5, and I say: five times 9, 45 apart from one fifth. Then, I say that, on that night, <the Moon> should shine 9 hours apart from 1 fifth, which is $^1/_5$, or 8 hours and 4 minutes, that is, $^2/_3\,^1/_{10}\,^1/_{30}$ one hour.

Par. The Moon waxes and wanes for 4 minutes per day, where a minute (λεπτόν) is in this case $^1/_5$ an hour (that is, it is $^1/_2$ of the minutes used in the previous sections); these 4 minutes amount to $^2/_3\,^1/_{10}\,^1/_{30}$ an hour. The algorithm for computing the duration of visibility v_a of the waxing and waning Moon at age of the Moon a (Rhabdas refers only to the time period from new Moon to full Moon, left unshaded below) is:

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$(a) \rightarrow$

$| \; 1 \leq a \leq 15, 4a \rightarrow 4a/5 = v_a$.

$| \; 16 \leq a, v_a = v_{30-a}$.

A computation is carried out for a Moon aged 11 days: the Moon shines 9 hours minus 1 minute, that is, 8 hours 4 minutes, which are $8\,{}^2\!/_3\,{}^1\!/_{10}\,{}^1\!/_{30}$ hours.

Comm. The first equality stated by Rhabdas is valid because ${}^2\!/_3\,{}^1\!/_{10}\,{}^1\!/_{30} = {}^{24}\!/_{30} = {}^4\!/_5$. The duration of visibility of the waxing (waning) Moon is supposed to increase (decrease) stepwise every day of a lunar month.[41] Accordingly, the visibility of the Moon within a cycle is approximated by a triangular step function. As the full Moon is supposed to “shine” for the length of the interval between sunset and moonset, the step is ${}^4\!/_5$ an hour, which is the scaling factor between 12 hours (the length of any night in seasonal hours) and 15 days.

Seasonal and equinoctial hours

But this algorithm will prove true only on the occasion of the equinox—if, on the same footing, the hours are conceived as seasonal all time along, viz. both the night and the day are also conceived of 12 hours and we conceive some <hours> greater and some lesser, but if the hours are reckoned as equinoctial on any occasion, that is, to be of equal length both the nocturnal and the diurnal hours, as when, grant that, the longest day is of 15 hours and the shortest night is of 9 hours, and inversely the longest night is of 15 hours and the shortest day is of 9 hours—this algorithm will not always prove true, but we shall always need another one. God willing, this <algorithm> from my own will also be expounded, and we shall not fear any rebutter.

It is as follows. Always multiply the age of the Moon that has been found by the <length in equinoctial> hours of that night on the occasion on which the search occurs, and divide the gathered number by the times of the equinoctial hour, which are 15, and conceive one hour for each pentadecad.

For example, let a search have occurred for us to find, in the month of January (on which occasion the night has 14 hours), how many hours should the Moon shine, its age

[41] For this algorithm, see *Anonymus* 892, sect. 25; *Anonymus* 1092B, sect. 5, in Karnthaler, “Die chronologischen Abhandlungen” (cit. n. 30), 9.159-170; *Anonymus* 1377, sect. 7, in *PG* XIX 1324–1328, where the algorithm is also described in detail. Latin computistic treatises include Bede, *De Temporum Ratione* XXIV and the Computus printed in *PL* CXXIX, 1305. The connection with Western sources is also made explicit in Theophilaktos’ unpublished Computus in Hamb., SUB, in scrin. 50a (*Diktyon* 32373), f. 11v (μάθημα τοῦ ψήφου τῶ<ν> Λατίνω<ν> ἑρμινευθὲν παρὰ τοῦ ἐλαχίστου Θεοφυλάκτου), whose last section expounds the same algorithm. See also Neugebauer, *HAMA* (cit. n. 29), 830, and Neugebauer, *Ethiopic Astronomy* (cit. n. 19), 176-177.

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being twelve days. And I say according to the expounded algorithm: twelve times 14, 168; these, once divided by 15, make 11 hours and $^1/_5$. And I say that, on that night, the Moon shines 11 hours and $^1/_5$; with the other algorithm expounded above, on the contrary, $9^1/_2$ hours and a tenth are found.

Further, one must also exemplify on the occasion on which the night has 9 hours, that is, in the month of June, how many hours should the Moon shine, its age being found fourteen days. And I also say anew according to the given algorithm: I multiply the 14 days of the Moon by the 9 hours of the night; and they yield 126, for nine times 14 make 126; then, I can remove 15 from these eight times; there also remain 6, which are the part two-fifths of 15, that is, the part a-third and $^1/_{15}$ of an hour. And I declare that, on that night, the Moon shines $8^1/_3{}^1/_{15}$ equinoctial hours. So much for you about the algorithms for the Moon too.

Par. This computation employs seasonal hours. The difference between seasonal hours and equinoctial hours is clarified. An algorithm that employs equinoctial hours, where N_X = length in equinoctial hours of the night in month X, is as follows (at equinox, $N_X = 12$, and this formula and the one given above coincide):

$(a,X) \rightarrow aN_X \rightarrow aN_X/15 = \nu_a$.

A computation carried out for a Moon aged 12 days in January (the night lasts 14 equinoctial hours) demonstrates that the Moon shines $11^1/_5$ hours, whereas the other algorithm (and thus reckoning by seasonal hours) gives $9^1/_2{}^1/_{10}$ hours. A computation is carried out for a Moon aged 14 days in June (the night lasts 9 equinoctial hours): the Moon shines $8^1/_3{}^1/_{15}$ hours.

Comm. A day is divided into 24 hours (or into 12 double-hours). These can be evenly distributed between the two complementary portions of a day determined by sunrise and sunset, in which case they are of variable length and are called “seasonal” (καιρικαί) hours. Alternatively, the hours can be of fixed length, namely, $^1/_{24}$ a nychthemeron; in this case, they are called “equinoctial” (ἰσημεριναί) hours, because this is the value of a seasonal hour at the equinoxes. The length of the night in equinoctial hours is traditionally approximated, at the latitude of Constantinople, by a linearly increasing step function ranging from 12 hours (in March and September) to 15 hours (in December) and back to 9 hours (in June).

10

Ῥητέον δὲ ἤδη καὶ περὶ τοῦ νομικοῦ Φάσκα, τουτέστι τοῦ Ἰουδαικοῦ, ὅπερ εὑρήσεις οὕτως. ἑνδεκαπλασίασον τὸν κύκλον τῆς σελήνης οἷος ἐστὶν ἀπὸ τοῦ πρώτου μέχρι αὐτοῦ τοῦ[a] ἐννακαιδεκάτου [[καὶ αὐτοῦ]]· εἶτα πρόσθες τῷ τοιούτῳ πολλαπλασιασμῷ καὶ ς, ἅς φασὶ τῶν αἰώνων, τουτέστι τῶν παρελθουσῶν ἓξ χιλιάδων τῶν χρόνων – ταύτας δὲ δεῖ προστιθέναι ἐν μόνοις τοῖς ιε κύκλοις τῆς σελήνης, ἤγουν ἀπὸ τοῦ α[ου] κύκλου τῆς σελήνης μέχρι τοῦ ἑξκαιδεκάτου· ἐν δὲ τοῖς λοιποῖς τέσσαρσι κύκλοις, εἴτουν ἐν τῷ

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ις^ω^ ιζ^ω^ ιη^ω^ καὶ ιθ^ω^ κύκλῳ, ἀντὶ τῶν ς πρόστιθε ζ – καὶ συναθροίσας πάντα ὁμοῦ |66v ἄφελε ἐξ αὐτῶν ὅσας εὕρῃς τριακοντάδας, καὶ τὰ κάτωθεν αὐτῶν ἐναπολειφθέντα κράτησον, καὶ ἐπίβαλον τούτοις ἀπαρχῆς τοῦ Μαρτίου μηνὸς ἡμέρας ὅσας χρῄζεις εἰς ἐκπλήρωσιν ἡμερῶν ν· εἰ δ᾽ οὐ σώσει ὁ Μάρτιος ὅλος, ἔπαρον τὰς ἐπιλοίπους καὶ ἀπὸ τὸν Ἀπρίλλιον, καὶ ἔνθα ἂν φθάσῃ πληρωθῆναι ὁ ἀριθμὸς τῶν ν ἡμερῶν, εἴτε ἐν τῷ Μαρτίῳ εἴτε ἐν τῷ Ἀπριλλίῳ, ἐν ταύτῃ λέγε γενέσθαι καὶ τὸ Φάσκα τὸ νομικόν.

Ὑποδείγματος δὲ χάριν ἐγένετο παρά τινων ζήτησις εὑρεῖν ἡμᾶς τὸ τοιοῦτον Φάσκα κατὰ τὸ νῦν ἐνεστὸς ,ςων ἔτος, καὶ ποιοῦμεν οὕτως κατὰ τὴν δεδομένην ἡμῖν μέθοδον, καὶ λέγομεν· εὑρέθη ὁ τῆς σελήνης κύκλος ἐν τῷ παρόντι καιρῷ ι, καὶ μετροῦμεν αὐτὸν ἑνδεκάκις· καὶ γίνεται ὁ ἀριθμὸς ρι· τούτοις προστίθεμεν καὶ ς· καὶ γίνονται ρις· ἀφαιρῶ ἐξ αὐτῶν τρεῖς τριακοντάδας· ἐναπελείφθησαν καὶ κς· χρῄζω οὖν εἰς ἐκπλήρωσιν [[καὶ]] τῶν ν ἡμερῶν ἡμέρας κδ, ἃς καὶ λαμβάνω ἀπὸ τοῦ Μαρτίου. λέγω οὖν εὑρεθῆναι τὸ νομικὸν Φάσκα κατὰ τὴν κδ τοῦ Μαρτίου μηνός.

[a] e corr. ^αὐ^τοῦ ^τοῦ^

Passover

Now one must also speak about Passover, namely, the one of the Jews, which you will find as follows. Undecuple the cycle of the Moon what<ever> it is, from the first one up to and including the nineteenth one; then add 6, which they call "of the eras", that is, of the bygone six thousands of years, to such a multiplication too—however, one must add these only in 15 cycles of the Moon, namely, from the 1st cycle of the Moon up to the sixteenth; in the remaining four cycles, viz. in the 16th, 17th, 18th, and 19th cycle, add 7 instead of 6—and putting all together remove as many thirties as you find from them, and keep that which is left down from them, and cast upon these, from the beginning of the month of March, as many days as you need for filling 50 days; if, however, the whole March is not enough, raise the remaining <days> from April too, and wherever the number of the 50 days happens to be filled first, whether in March or in April, say that Passover also occurs there.

For example, let a search by some people have occurred for us to find Passover in the now-present year 6850, and we do as follows according to the algorithm I have given, and we say: on the present occasion, the cycle of the Moon has been found to be the 10th, and we measure it eleven times; and it yields number 110; we also add 6 to these; and they yield 116; I remove three thirties from them; 26 are also left out; then, I need 24 days for filling 50 days, which I also take from March. Then, I say that Passover has been found on the 24th of the month of March.

Par. The algorithm for finding the date p_m of Passover (τὸ νομικὸν Φάσκα, Φασκάλιον) at lunar cycle year m is (see below for an explanation of the shading):

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$(m) \rightarrow 11m \rightarrow$

$| \; m < 16, 11m + 6 \rightarrow (11m + 6) \bmod 30 \rightarrow 50 - [(11m + 6) \bmod 30] -: 1_M = p_m$.

$| \; 16 \leq m \leq 19, 11m + 7 \rightarrow (11m + 7) \bmod 30 \rightarrow 50 - [(11m + 7) \bmod 30] -: 1_M = p_m$.

The 6 units to be added to $11m$ are the "epacts of the eras" (ἐπακταὶ τῶν αἰώνων), that is, of the 6 whole millennia elapsed since Creation. The counting of the days begins on March 1 but it might end in April. A computation is carried out for current year AM 6850 [= 1342], and it yields $m = 10 \rightarrow p_m = 24_M$.

Comm. The first branch of the algorithm can be described as follows: multiply the lunar cycle year m by 11, add 6 units, reduce modulo 30, subtract the result from 50 and count as many days as the remainder from March 1: the resulting day is the date of Passover; this day falls in April if the remainder is greater than 31. This widespread algorithm is elsewhere called "notarial" (νοταρική).[42] The addendum $11m$ is the age of the Moon at the end of lunar cycle year $m - 1$ (mod 19), that is, it is its epacts. This algorithm simplifies the fundamental algorithm expounded in early sources such as Heraclius and George Presbyter[43] and which is actually a pretty straightforward adaptation to the Byzantine era of the algorithm adopted in the early Alexandrian Church. Heraclius' and George's algorithm has by far more complex branching conditions and prescribes subtracting from 44, not from 50 (44 is the number of days nearest to 1 and a half lunar month, as $29\frac{1}{2} + 14\frac{1}{2}\frac{1}{4} = 44\frac{1}{4}$). The said simplification of these early algorithms was carried out by writing 44 as the result of 50 – 6, with the parameter 50 lying outside the modulo 30 reduction and the parameter 6 lying inside it: this rewriting allowed setting a branching condition much more transparent than the condition in Heraclius' and George's algorithm. This can be so explained: counting 50 days starting on March 1 one gets to April 19, which is the upper term for Passover (see sect. **13**), hence no counting from April 1 is required

[42] By *Anonymus* 1079, sect. 5, in Mentz, "Beiträge" (cit. n. 38), 98. Other occurrences of this algorithm are in *Anonymus* 892, sect. 12; *Anonymus* 1092A, sect. 4, and 1092B, sect. 6, in Karnthaler, "Die chronologischen Abhandlungen" (cit. n. 30), 5.40-6.47 and 9.191-10.198, respectively; *Anonymus* 1079, sect. 5, in Mentz, "Beiträge" (cit. n. 38), 100; *Anonymus* 1183, sect. 6; *Anonymus* 1247, sect. 3, in Schissel, "Chronologischer" (cit. n. 30), 106; *Anonymus* 1256, sect. 6; Matthew Blastares, in Rhalles, Potles, Σύνταγμα (cit. n. 30), 416; *Anonymus* 1377, sect. 5, in *PG* XIX, 1328; *Anonymus* 1379, in *PG* XIX, 1329.

[43] See H. Usener, "De Stephano Alexandrino", in Idem, *Kleine Schriften*, III, Leipzig – Berlin 1914, 311-317, sect. 30; Diekamp, "Der Mönch" (cit. n. 34), sect. 4 on 30-31, and the discussion in Acerbi, "Byzantine Easter Computi" (cit. n. 19). See also the analyses in A. Tihon, "Le calcul de la date de Pâques de Stéphanos-Héraclius", in B. Janssens, B. Roosen, P. Van Deun (eds.), *Philomathestatos. Studies in Greek and Byzantine Texts Presented to Jacques Noret for his Sixty-Fifth Birthday* (*Orientalia Lovaniensia Analecta* 137), Leuven 2004, 625-646 (Heraclius), and J. Lempire, "Le calcul de la date de Pâques dans les traités de S. Maxime le Confesseur et de Georges, moine et prêtre", *Byzantion* 77 (2007), 267-304 (George).

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for large epacts, contrary to what is done in Heraclius' and George's algorithm. The 6 units to be added to 11*m* are called, in Rhabdas' work as well as in other Computi,[44] "epacts of the bygone eras" (ἐπακταὶ τῶν αἰώνων παρελθόντων), which correspond to the 6 whole millennia elapsed since Creation: this is the basic mnemonic trick in this computation of Passover. In the second branch of the algorithm, the additional unit to be added to 11*m* in the cycle years from 17 to 19 (and thus 7 units are added instead of 6) is the *saltus lunae*. Therefore, Rhabdas mistakenly locates the lunar cycle year starting from which 7 units must be added (his formulation is unambiguous): contrary to what he claims, in lunar cycle 16, 6 units must still be added. As usual, whole lunar months are removed by reducing modulo 30. Unnecessary complications would arise from reducing modulo $29\frac{1}{2}$; moreover, one would not let Easter coincide with Passover and reducing modulo 30 instead of modulo $29\frac{1}{2}$ shifts forward, and most conveniently, the schematic date of the computed Passover.

11

Ἄρτι χρῄζω μαθεῖν καὶ τὴν ἡμέραν ἐν ᾗ μέλλει γενέσθαι τὸ νομικόν, ὡς ἂν ἐξ αὐτῆς εὕρω καὶ τὸ ἡμέτερον εὐσεβὲς τῶν πιστῶν ἅγιον Πάσχα, καὶ εὑρίσκω ταύτην διὰ τῆς μεθόδου τοῦ ἡμεροευρεσίου οὕτως.

Κράτησον τὸν κύκλον τοῦ ἡλίου οἷος ἐστί, καὶ τούτῳ πρόσθες τὰ ἐπιβάλλοντα αὐτῷ τέταρτα, ἃ καὶ βίσεκτα λέγομεν, καὶ ἔκτοτε ἄρξου λαμβάνειν ἀπ᾽ ἀρχῆς τοῦ Ὀκτωβρίου μηνὸς [[λαμβάνειν]] ἀφ᾽ ἑνὸς ἑκάστου τῶν παρελθόντων μηνῶν, ἀπὸ μὲν τῶν ἐχόντων ἡμέρας λα, ἡμέρας γ, ἀπὸ δὲ τῶν ἐχόντων λ, ἡμέρας β, καὶ συναγαγὼν ὁμοῦ πάσας πρόσθες αὐτοῖς καὶ τὰς ἡμέρας τοῦ μηνὸς ἐκείνου ἐν ᾗ[a] τὴν ζήτησιν ποιεῖς, καὶ ἑνώσας ὁμοῦ ἄφελε ἐξ αὐτῶν πάσας τὰς ἑβδομάδας, καὶ τὰ κάτωθεν εὑρεθέντα ἀπὸ τῶν ζ, εἰ μὲν μία καταλειφθῇ, ἐστὶ κυριακή, εἰ δὲ δύο, δευτέρα, εἰ δὲ γ, τρίτη, καὶ ἑξῆς ὁμοίως μέχρι τῶν ζ.

Ὑποδείγματος χάριν ἔστω εὑρεῖν ἡμᾶς τὴν κδ τοῦ Μαρτίου μηνός (ἐν ᾗ καὶ τὸ νομικὸν ἔτυχε Φάσκα) ποία τῆς ἑβδομάδος ἡμέρα ἐστί, καὶ λέγομεν οὕτως. ὀκτωκαιδέκατος κύκλος τοῦ ἡλίου, τέσσαρες ἐπακταὶ αἱ ἀπὸ τῶν βισέξτων· Ὀκτωβρίου γ – λα γὰρ ἔχει – Νοεμβρίου β – λ γὰρ ἔχει – Δεκεμβρίου γ, Ἰαννουαρίου γ καὶ Μαρτίου κδ· ὁμοῦ ἡμέραι νζ, ἐξ ὧν ἀφαιρῶ ἑβδομάδας η· ἐναπελείφθη καὶ μία, ἥτις ἐστὶν ἡ τῶν βαίων κυριακή· καὶ ἡ ἐρχομένη ἑτέρα κυριακὴ ἔστιν ἡ λαμπρὰ τῆς ἀναστάσεως τοῦ κυρίου καὶ θεοῦ καὶ σωτῆρος ἡμῶν Ἰησοῦ Χριστοῦ ἡμέρα, ἐν ᾗ καὶ ἡμεῖς οἱ ὀρθόδοξοι Χριστιανοὶ τὸ ἡμέτερον εὐσεβὲς καὶ ἅγιον ἑορτάζομεν Πάσχα, ἤτοι τῇ λα τοῦ Μαρτίου μηνός.

Ἰστέον γὰρ ὅτι τῇ ἑβδομάδι ἐκείνῃ καθ᾽ ἣν τὸ νομικὸν γίνεται Φάσκα, τῇ κυριακῇ τῆς αὐτῆς ἑβδομάδος ἀμεταθέτως |67r ἐπιτελεῖται καὶ τὸ ἡμέτερον· εἰ δὲ τύχῃ κυριακῇ,

[44] See, for instance, *Anonymus* 892, sect. 12; *Anonymus* 1092A, sect. 4, in Karnthaler, "Die chronologischen Abhandlungen" (cit. n. 30), 5.42.

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ὥσπερ ἔτυχε νῦν, ἡ ἐπιοῦσα κυριακὴ ἔστιν ἡ τὸ ἡμέτερον ἀποδεικνύουσα· μετροῦμεν γὰρ τὰς ἐναπολειφθείσας τῆς ἑβδομάδος ἐκείνης ἡμέρας, καὶ προστίθεμεν τῇ ποστῇ τοῦ μηνὸς ἐκείνου ἐν ᾗ τὸ νομικὸν ἔτυχε, καὶ τὴν ποσωθεῖσαν ἡμέραν ἀπὸ τοῦ τοιοῦδε ἀριθμοῦ, ἐκείνην λέγομεν εἶναι τὴν ὑποδεχομένην τὴν τοῦ Χριστοῦ ἁγίαν ἀνάστασιν.

Ἵνα δὲ καὶ ἐπὶ ὑποδείγματος σαφέστερον γένηται τὸ λεγόμενον, εὑρέθη τῇ κδ τοῦ Μαρτίου [[τοῦ]] μηνὸς ἐν ἡμέρᾳ κυριακῇ τὸ νομικὸν Φάσκα· τῇ ἐπιούσῃ λοιπὸν ἐξ ἀνάγκης κυριακῇ γενήσεται καὶ τὸ ἡμέτερον. εἰσὶ γοῦν ἡμέραι ζ (ἤτοι ἑβδομὰς ὁλόκληρος), ἃς προστιθέντες τῇ κδ τοῦ Μαρτίου ποιοῦμεν λα· τῇ λα ἄρα τοῦ Μαρτίου ἐστὶ καὶ τὸ τῶν πιστῶν ἅγιον Πάσχα. ταύτῃ δὲ τῇ μεθόδῳ χρώμενοι τοῦ ἡμεροευρεσίου καὶ ἄλλην ὁποίαν ἐθέλεις ἡμέραν τοῦ ἐνιαυτοῦ εὑρήσεις ἀναντι<ρ>ρήτως.

[a] lege ᾧ

The weekday of an assigned date; Easter

I just need to learn the day on which Passover should also occur, so that I could also find our sacred holy Easter of the believers from it, and I find this by means of the day-finding algorithm, as follows.

Keep the cycle of the Sun what<ever> it is, and add to this the fourths cast upon it, which we also call "leap days", and thereafter begin taking, from the beginning of the month of October <and> from each single month among the bygone ones, 3 days from the months that have 31 days, 2 days from those that have 30 <days>, and gathering all of them together add to them the days of that month in which you do your search too, and uniting them together remove all weeks from them, and that which is found down from the 7s, if one <day> is left out, this is Sunday, if two, it is Monday, if 3, it is Tuesday, and similarly in succession up to 7.

For instance, let there be for us to find what weekday is the 24th of the month of March (on which Passover also falls), and we say as follows. Eighteenth cycle of the Sun, four epacts <arising> from the leap years; of October, 3—for it has 31 <days>—of November, 2—for it has 30—of December, 3, of January, 3, and of March, 24; together 57 days, from which I remove 8 weeks; one <day> is also left out, which is Palm Sunday; and the other, forthcoming Sunday is the bright day of the resurrection of our Lord and God and Saviour Jesus Christ, on which we orthodox Christians also celebrate our sacred and holy Easter, viz. on the 31st of the month of March.

For one must know that, in that week in which Passover occurs, our <Easter> is also invariably celebrated in the Sunday of the same week; if, however, <Passover> falls on a Sunday, as it falls now, the subsequent Sunday is the one which exhibits our <Easter>, for we determine the days left out in that week, and we add them to the date of that month

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in which Passover falls, and the day such a number amounts to, that one we say that it is the one that receives the holy resurrection of Christ.

In order for that which is said to become clearer by means of an example too, Passover has been found on the 24th of the month of March, a Sunday; consequently, our <Easter> will also occur by necessity on the subsequent Sunday. Now then, these are 7 days (viz. a whole week), which, once adding them to March 24, we make 31; therefore, the holy Easter of the believers is also on March 31. By using this day-finding algorithm you will also incontrovertibly find whatever other day of the year you want.

Par. Easter (Πάσχα) can be found once the weekday (ἡμέρα τῆς ἑβδομάδος) upon which Passover falls is ascertained. To this end, the following general day-finding algorithm (ἡμεροευρέσιος μέθοδος) for the weekday $w(x_X)$ of day x of month X in a given solar cycle year s (n_k = number of days in month k) is needed:

$(s,x,X) \rightarrow s + [\![s/4]\!] \rightarrow s + [\![s/4]\!] + \sum_{k=0}^{X-1}(n_k - 28) \rightarrow s + [\![s/4]\!] + \sum_{k=0}^{X-1}(n_k - 28) + x \rightarrow$

$\rightarrow [s + [\![s/4]\!] + \sum_{k=0}^{X-1}(n_k - 28) + x] \bmod 7 = w(x_X)$.

Weekdays from Sunday to Saturday are numbered from 1 to 7.

A computation is carried out for current year AM 6850 [= 1342] March 24 (Passover), and it yields $s = 18 \rightarrow w(24_M) = 1$, which means that Passover falls on Palm Sunday. Therefore, the day on which the Orthodox Church celebrates Easter is the following Sunday. An algorithm for finding Easter as the Sunday which comes next a given date is:

$(p_m) \rightarrow p_m + [8 - w(p_m)] \rightarrow \{p_m + [8 - w(p_m)]\} \bmod 31 = r_m$.

A computation is carried out for current year AM 6850 [= 1342], and it yields $p_m = 24_M \rightarrow r_m = 31_M$.

Comm. The algorithm computes the weekday of any date x in month X.[45] To this end, it suffices to count the days elapsed from a date falling on a known weekday and remove whole weeks. It should be kept in mind that a year of 365 days exceeds a whole number of weeks by 1 day (summand s in the above algorithm: recall that the Byzantine world era and the solar cycle are synchronized; this summand also includes 365 of the 366 days of a leap year), a leap year exceeds it by 1 additional day (further summand $[\![s/4]\!]$),[46] a month exceeds it by its own length in days minus 28 days (= 4 weeks), namely, $n_k - 28$ in our notation. The sum $\sum_{k=0}^{X-1}(n_k - 28)$ is the excess over 28 days of the months from October to the one preceding the given month X. The

[45] This algorithm is ubiquitous in Byzantine Computi. See *Anonymus* 892, sect. 12; *Anonymus* 1079, sect. 1, in Mentz, "Beiträge" (cit. n. 38), 76; Psellos, sect. I.13, in Redl, "La chronologie" (cit. n. 36), II, 229-232; *Anonymus* 1183, sect. 7.

[46] Solar cycle years are used only in computistical algorithms of this kind.

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date x must then be added. Reducing the sum modulo 7 involves eliminating whole weeks. As only months of 31 and 30 days are mentioned and because of the leap year contribution included in the term $[\![s/4]\!]$, February must be set to 28 days; given the fact that the summand $[\![s/4]\!]$ is operative throughout the year, the month X must be a month coming after February. This restriction, however, is of no consequence as far as Passover or Easter computations are concerned. To check the consistency of the algorithm, we should recall that the weekday of the epoch date of the Byzantine world era is a Saturday = 7, so that $w(1_O) = 2$ for the first day of the solar cycle, which is the output for $s = 1$, $x = 1$.

12

Ἡ δὲ τοῦ βισέξτου εὕρεσις τοιαύτην ἔχει τὴν αἰτίαν. τοῦ ἡλίου τὸν οἰκεῖον {κύκλον} διὰ τξε νυχθημέρων καὶ δ^ον ἑνὸς νυχθημέρου, ὅπέρ ἐστιν ὧραι ς, ἀνύοντος κύκλον, κατὰ τέσσαρας ἐνιαυτοὺς ἀπαρτίζεται νυχθήμερον ἕν, ὃ δὲ καὶ προστίθεται τῷ Φευρουαρίῳ μηνὶ ὡς ἐλλειπεῖ ὄντι – ἔχει γὰρ ἡμέρας κη – καὶ τότε δεχόμενον καὶ τοῦτο ἔχει ἡμέρας κθ, καὶ γίνεται ὁ χρόνος ἐκεῖνος νυχθημέρων τξς.

Ὅτε οὖν ἐθέλεις εἰδέναι εἴτε βίσεξτος ἔνι ὁ χρόνος εἴτε καὶ μή, κράτησον τὰ ἀπὸ κτίσεως κόσμου εὑρισκόμενα ἔτη, καὶ ὕφειλον αὐτὰ ἐπὶ τῶν δ, κἂν μὲν οὐδέν τι καταλειφθῇ, βίσεξτος πάντως ὁ χρόνος ἐστίν· εἰ δὲ ἓν ἢ δύο ἢ τρία καταλειφθῶσιν, οὐκ ἔστι βίσεξτος ὁ ἐνιαυτός.

Ἢ καὶ ἄλλως διὰ τὸ σύντομόν τε καὶ ῥᾴδιον. κράτει τὰ κάτωθεν τῶν ,ςω ἐτῶν εὑρισκόμενα ἔτη, ἅπέρ εἰσι νῦν κατὰ τὸ παρόντα χρόνον ν, καὶ ταῦτα δίελε ὁμοίως εἰς δ, τουτέστιν ἔκβαλε πάσας τὰς τετράδας, κἂν μὲν οὐδέν τι καταλειφθῇ ἀλλ᾽ εἰς δ τελευτήσει, βίσεξτος ὁ χρόνος ἐστίν· εἰ δὲ ἓν ἢ β ἢ γ, ὡς εἴρηται, βίσεξτον[a] οὐκ ἔστιν.

Ὑποδείγματος δὲ χάριν ἐζητήθη νῦν καὶ κατὰ τὸ παρὸν ,ςων ἔτος εὑρεῖν ἡμᾶς τὸν χρόνον εἴτε βίσεξτον ἔχει εἴτε καὶ μή. καὶ ἀναλύωμεν τὰ ἔτη εἰς δ οὕτως, καὶ λέγομεν· τετράκις τὰ ,α, ,δ· τετράκις τὰ φ, ,β· δ^ις τὰ σ, ω· τετράκις τὰ ιβ, μη· ἐναπελείφθησαν καὶ β. καὶ λέγομεν ὅτι ὁ χρόνος οὐκ ἔχει βίσεξτον. βίσεξτον δὲ λέγομεν τὸ διὰ τῶν δ ἐνιαυτῶν ἀποτελούμενον νυχθήμερον· βίσεξτον δὲ εἴρηται ἀπὸ τοῦ φάναι τὸν ἱερέα τὸ παλαιὸν κατὰ μόνον τὸν Φευρουάριον μῆνα δὶς πρὸ ἓξ καλανδῶν. τοσαῦτα σοι περὶ τῆς εὑρέσεως τοῦ βισέξτου καὶ τῆς ἡμέρας. ἀλλ᾽ ἐπανιτέον ἡμῖν τὸν λόγον ὅθεν καὶ κατελήξαμεν τὸ λεῖπον ἀναπληροῦντες τοῦ πασχαλίου.

[a] lege βίσεξτος

Leap days and leap years

The finding of the leap day has the following rationale. As the Sun passes through its own circle in 365 nychthemera and $^1/_4$ of a nychthemeron, which are 6 hours, every four years

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it completes one nychthemeron, which is also added to the month of February because this is defective—for it has 28 days—and once it has also received this <nychthemeron> it has 29 days, and that year becomes of 366 nychthemera.

Then, whenever you want to know whether a year is a leap year or not, keep the years found from the foundation of the world, and remove them by 4, and whenever nothing is left out, the year is always a leap year; if one or two or three are left out, the year is not a leap year.

Or also in another way, for the sake of both conciseness and easiness. Keep the years found down from 6800 years, which are 50 now in the present year, and similarly remove these by 4, that is, cast all tetrads aside, and whenever nothing is left out but it ends in 4, the year is a leap year; if <it ends in> one or 2 or 3, as said, it is not a leap year.

For example, let there have been a search for us to find, now and in the present year 6850, whether the year has a leap day or not. And we resolve the years out into 4 as follows, and we say: four times 1000, 4000; four times 500, 2000; 4 times 200, 800; four times 12, 48; 2 are also left out. And we say that the year does not have a leap day. We call the nychthemeron completed every 4 years “bissextile”; this is said “bissextile” from the fact that in early times the priest could speak only in the month of February, on twice-sixth before Calends. So much for you about the finding of the leap year and of the <leap> day. But we must return to the point on which we have also stopped our exposition, by completing what remains of the paschalion.

Par. The Sun traverses its own circle in $365\frac{1}{4}$ days, therefore every 4 years the 6 exceeding hours make 1 full day, which is added to February. The resulting leap year (βίσεξτος) comprises 366 days.

The criterion for identifying whether a year is a leap year or not is: if $y \equiv 4 \pmod{4}$, then y is a leap year.

A concise and easy criterion is: if $y - 6800 \equiv 4 \pmod{4}$, then y is a leap year.

A computation is carried out for current year AM 6850 [= 1341/2], which is not a leap year because $6850 \equiv 2 \pmod{4}$.

A βίσεξτος “leap day” or “bissextile” is the nychthemeron completed every four years. The origin of the denomination “bissextile” is as follows: the ancient priest was allowed to speak only on a “bis-sextus” (twice-sixth) day before the March Kalends.

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|67v Ἐξότου τοίνυν γνωρίσεις ἐν ποίῳ μηνὶ ἐγένετο τὸ Πάσχα καὶ ἐν πόστῃ τούτου ἡμέρᾳ, καὶ μέλλεις εὑρεῖν καὶ τὴν Ἀπόκρεω εἰς πόστην τοῦ Ἰαννουαρίου ἡμέραν ἢ τοῦ

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Φευρουαρίου μέλλει γενέσθαι, εἰ μὲν οὐκ ἔστιν ὁ χρόνος βίσεξτος, ἀεὶ πρόστιθε τῇ ποστῇ τοῦ μηνὸς ἐκείνου ἐν ᾗ ἔτυχε τὸ Πάσχα ἡμέρας γ, εἰ δὲ ἔνι βίσεξτος, δ, καὶ κατʼ ἐκείνην τὴν ποσότητα τοῦ Ἰαννουαρίου ἢ τοῦ Φευρουαρίου ἴσθι γίνεσθαι καὶ τὴν Ἀπόκρεω. ἰστέον δὲ ὡς ἐὰν εὑρεθῇ τὸ Πάσχα ἀπὸ τῆς εἰκοστῆς δευτέρας τοῦ Μαρτίου μέχρι καὶ τῆς εἰκοστῆς ὀγδόης τοῦ αὐτοῦ, πάντοτε εἰς τὸν Ἰαννουάριον μῆνα γίνεται ἡ Ἀπόκρεω· ἀπὸ δὲ τῆς κη καὶ ἑξῆς ἄχρι τῆς κε τοῦ Ἀπριλλίου ἀεὶ εἰς τὸν Φευρουάριον εὑρίσκεται.

ἰστέον σοι δὲ καὶ τοῦτο, ὡς οὐδέποτε γίνεται τὸ Πάσχα κάτωθεν τῆς κβ τοῦ Μαρτίου μηνὸς ἀλλʼ οὐδὲ ἄνωθεν τῆς κε τοῦ Ἀπριλλίου. ὡσαύτως καὶ τὸ νομικὸν οὔτε ἔσωθεν τοῦ κα τοῦ Μαρτίου γίνεται οὔθʼ ὑπερβάλλει τὴν τοῦ Ἀπριλλίου ὀκτωκαιδεκάτην, ὅτι δὲ τοῖς μὲν Ἑβραίοις παραδέδοται θύειν τὸν ἀμνὸν κατὰ τὴν πανσέληνον τοῦ πρώτου μηνὸς τοῦ παρʼ αὐτοῖς καλουμένου Νισὰν ἐντὸς τῆς ἐαρινῆς ἰσημερίας, ἡμεῖς δὲ παρευθὺς τῇ γειτνιαζούσῃ κυριακῇ.

ἔστιν ὅτε καὶ παρατρέχομεν ἡμεῖς ἑβδομάδα μίαν διὰ τὸ μὴ γίνεσθαι τὴν πανσέληνον ἐν τῷ παραδοθέντι τοῦ νομικοῦ ἀριθμῷ ἀνέκαθεν καὶ ἐξ ἀρχῆς παρὰ τῶν θεσπεσίων καὶ ἁγίων πατέρων ἡμῶν, τῆς σελήνης τὴν αἰτίαν ἐχούσης ἐκ τῆς τοῦ παντὸς μεταποιήσεως· μετακινεῖσθαι γὰρ λέγουσι τὴν μεγίστην σφαῖραν οἱ τῶν ἀστρονόμων δεινότεροι μοῖραν μίαν κατʼ ἐνιαυτοὺς ἑκατόν· οἱ δὲ τούτῳ[a] μεταγενέστεροι ἀκριβέστερον τὰς τηρήσεις ποιούμενοι καὶ μέχρι τῶν ο λέγουσιν. ἦν μὲν ῥᾴδιον καὶ ἡμῖν τὴν τούτου γενέσθαι διόρθωσιν ὑποτεταγμένων ἁπάντων τῶν Χριστιανῶν ὑφʼ ἑνὶ δεσπότῃ καὶ βασιλεῖ, ὥσπερ ἦν ἐν τῇ τοῦ μεγάλου Κωνσταντίνου ἀρχῇ· νῦν δὲ μὴ οὕτως ὄντος ἄτακτον δοκεῖ ἡμῖν μὲν ἐν ἄλλῃ ἡμέρᾳ ποιοῦσι τὸ Πάσχα, ἐν ἄλλῃ δὲ τοὺς Ἰταλοὺς καὶ τοὺς Ἴβηρας ἔτι τε Τριβαλούς, Βουλγάρους, Ῥώσους, Ἀλανούς, Ζίκχους καὶ τὰ λοιπὰ γένη τῶν Χριστιανῶν. καὶ διὰ ταύτην τὴν αἰτίαν ἐστέρχθη γίνεσθαι ὡς γίνεται διὰ τὸ τῆς ὁμονοίας καὶ τῆς εὐταξίας καλόν, ἵνα μὴ δόξῃ παρὰ τοῖς ἄλλοις καινοτομίαν τινὰ καὶ σύγχυσιν γίνεσθαι. οὐκ ἀεὶ δὲ πάντως καὶ καθʼ ἕκαστον ἐνιαυτὸν ἑβδομάδα τῷ νομικῷ παρατρέχομεν, ἀλλὰ σπανίως καὶ ἔν τισι καιροῖς, ὥσπερ δῆτα καὶ νῦν ἔτυχε· τῆς[b] γὰρ ψήφου τὴν κδ τοῦ Μαρτίου τὸ νομικὸν ἡμῖν παραδηλοῦντος ἐν ἡμέρᾳ κυριακῇ, τῆς πανσελήνου τοῦ πρώτου παρʼ Ἑβραίοις μηνὸς μὴ οὔσης ἐν αὐτῇ ἀλλʼ ἐν τῇ κβ τοῦ Μαρτίου τῇ παρασκευῇ τῶν βαίων, οὐκ ἐτελέσαμεν ἡμεῖς τὸ Πάσχα τῇ κδ τοῦ Μαρτίου ἀλλὰ κατὰ τὴν λα τοῦ Μαρτίου διὰ τὰς προλαβούσας \αἰτίας/. καὶ ταῦτα σοι δὲ διεξῆλθον, ἵνα μὴ παντελῶς ἀμύητος τῶν τοιούτων εἴης.

|68r Ὑποδειγματιστέον σοι διὰ τὸ σαφέστερον καὶ τὸ ....τερον. εὑρέθη τὸ Πάσχα κατὰ τὸ παρόντα καιρὸν εἰς τὴν λα τοῦ Μαρτίου μηνός, καὶ ζητῶμεν μαθεῖν τοῦ μέλλει εὑρεθῆναι ἡ Ἀπόκρεω. ἐπεὶ γοῦν οὐδὲν ἔνι ὁ χρόνος βίσεξτος, προστίθημι τῇ λα τοῦ Μαρτίου γ· καὶ γίνονται ὁμοῦ λδ· ἐκ τούτων ἀφαιρῶ τὴν λα ἡμέραν τοῦ Ἰαννουαρίου· καὶ ἐναπελείφθησαν γ, αἵτινες ἐμπίπτουσιν εἰς τὸν Φευρουάριον· ὅταν γὰρ ἐν τούτῳ ὑπερβῇ ὁ ἀριθμὸς τὴν λα, ἐκβαλλομένης αὐτῆς τὸ καταλειφθὲν δέχεται ὁ Φευρουάριος. εὑρέθη οὖν ἡ Ἀπόκρεω εἰς τὰς γ τοῦ Φευρουαρίου μηνός.

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[a] fort. lege τούτοις [b] exp. τοῦ cf. παραδηλοῦντος

Meat-Fare Sunday

Now then, as soon as you recognizes in what month and on what date of it Easter has occurred, and you should also find on what date of January or of February Meat-Fare should occur, if the year is not a leap year, always add 3 days to the date of that month in which Easter falls, if it is a leap year, 4, and know that Meat-Fare also occurs on that date of January or of February. One must know that, whenever Easter is found from the twenty-second of March up to and including the twenty-eighth of the same <month>, Meat-Fare always occurs in the month of January; <whenever Easter is found> from <March> 28 and successively up to April 25, <Meat-Fare> is always found in February.

Par. The algorithm for finding the date t of Meat-Fare Sunday (Ἀπόκρεω) being r the date of Easter is as follows:[47]

$(r,y) \to r + 3 + [\![(y \bmod 4)/4]\!] \to r + 3 + [\![(y \bmod 4)/4]\!] -: 1_J = t$.

If $22_M \leq r \leq 28_M - [\![(y \bmod 4)/4]\!]$, then $t \in J$; if $29_M \leq r \leq 25_A$, then $t \in F$.

Comm. In the Byzantine liturgical calendar, Meat-Fare is the third Sunday of the pre-Lenten period of preparation and repentance;[48] it falls 8 weeks = 56 days before Easter. As in non-leap years February plus March last 59 days, Meat-Fare Sunday falls numerically 3 days after Easter (summand $r + 3$) but 2 months before the month in which Easter falls, with the due adjustment in leap years (summand $[\![(y \bmod 4)/4]\!]$), and keeping in mind that if the date of Easter falls on a day after March 29 (28 in leap years), then Meat-Fare Sunday falls in February rather than in January. A modulo 31 reduction is not envisaged by Rhabdas, but it must be introduced in order to take into account Easter dates in March shifting to April because of the addition of $3 + [\![(y \bmod 4)/4]\!]$. See also the end of this section.

Easter and Passover Terms

You must also know this, that Easter never occurs below the 22nd of the month of March nor, on the other hand, above April 25. Likewise, Passover neither occurs within March 21 nor does it exceed the eighteenth of April, because the Jews traditionally slay the lamb

[47] The floor function is particularly effective in formalizing leap year computations. In fact, if y is a year in the Byzantine world era or in the era AD, $[\![(y \bmod 4)/4]\!]$ singles out leap years—which in both eras are such that $y = 4k$ for some integer k—because $y \equiv 1, 2, 3$ or $4 \pmod 4$, and $[\![{}^1\!/_4]\!] = [\![{}^2\!/_4]\!] = [\![{}^3\!/_4]\!] = 0$, $[\![1]\!] = 1$. As taking the floor of a division involves taking its integer quotient by disregarding the remainder, $[\![y/4]\!]$ is the total number of leap years since epoch.

[48] See *Anonymus* 892, sect. 12; *Anonymus* 1079, sect. 2, in Mentz, "Beiträge" (cit. n. 38), 78; *Anonymus* 1183, sect. 8; *Anonymus* 1256, sect. 9; Matthew Blastares, in Rhalles, Potles, Σύνταγμα (cit. n. 30), 418; Isaac Argyros, in *PG* XIX, 1301 and 1304.

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on the full Moon of the first month, the one they call "Nisan", <which begins> within the Spring equinox, whereas we <celebrate our Easter> on the immediately adjacent Sunday.

Par. The terms for Easter are: $22_M \le r \le 25_A$. The terms for Passover are: $21_M \le p \le 18_A$. The Jews slay the lamb on the full Moon of the lunar month that includes the Spring equinox (ἐαρινὴ ἰσημερία), a month they call "Nisan"; the Christians celebrate Easter on the Sunday next after Passover.

Comm. The intervals given above set the standard terms for Easter and Passover in Byzantine Computi; the former terms straightforwardly derive from the latter by applying the rules for finding the date of Easter. The Passover terms are so determined because the Spring equinox (March 21) is the lower bound and Passover can fall within 1 lunar month from that date. The real upper bound for Passover is April 19: see note 22 above, and the commentary on sect. **15**.

Discrepancies between the actual full Moon and the Passover date

Sometimes we also overrun a single week because the full Moon does not occur on the numerical date for Passover handed down in the first place and originally by our divine and holy Fathers, the Moon inheriting the cause <of this> from the transformation of the entire <Cosmos>; for the most venerable astronomers say that the greatest sphere revolves by one degree every hundred years, whereas those later than them, who performed more accurate observations, claim that <it revolves> no more than up to 70 <degrees every 100 years>. It would also be easy for us to carry out a correction of this <discrepancy>, were all Christians subjected to one single ruler and king, very much as this was the case during the realm of Constantine the Great; now, since this is not the case, we disorderly deem it fit to set Easter on a specific day, whereas the Latins and the Iberians set it on another, and again the Triballi, the Bulgars, the Russians, the Alans, the Zicchi, and all the remaining Christian denominations. And for this reason, owing to the virtues of concord and of orderliness, one is content with perpetuating this state of affairs, in order for the others not to believe that there occurs any changing for the sake of changing and confusion. However, it is not the case that we overrun a week from Passover always and every year, but seldom and on specific occasions, exactly as it also happened to be the case right now; for, as the calculation intimates for us that Passover <falls> on March 24, a Sunday, whereas the full Moon of the first Jewish month does not fall on it but on March 22, the Friday before Palm Sunday, because of the previously-adduced reason we did not celebrate Easter on March 24, but on March 31. And go carefully through these <arguments>, in order not to be altogether uninitiated to them.

One must set out an exemple for you, for the sake of greater clarity and ***. On the present occasion, Easter has been found on the 31^{st} of the month of March, and let us seek to learn <where> should Meat-Fare be found. Now then, since the year is by no way a leap year, I add 3 to March 31; and together they yield 34; I remove the 31 days of January from

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these; and 3 are left out, which fall in February; for whenever in this <computation> the <resulting> number oversteps 31, casting it [*scil.* 31] aside February receives what is left out. Then, Meat-Fare has been found on the 3rd of the month of February.

Par. Occasional discrepancies between the actual full Moon (and therefore the actual Passover) and the Passover date that is computed according to the prescriptions of the Church Fathers result in more than 1 week of interval between the full Moon (that is, the actual Passover according to the Jews) and Easter; the reason adduced is the precession of the equinoxes (1 degree per century according to Ptolemy, 1 degree every 70 years according to later astronomers, who relied on more accurate observations).[49] Despite the fact that homogeneous standards were achieved when all Christians were united under a single ruler, for instance during Constantine the Great, occasional discrepancies were thereafter left unsettled in order to avoid creating even more confusion, as in Christendom the date of Easter was variable according to the Christian denominations. The discrepancies between the actual full Moon and the Passover date are occasional; notably, an instance occurs in the year in which Rhabdas writes, as the full Moon occurred on March 22,[50] whereas computations for Passover yield March 24.

A computation is carried out for current year AM 6850 [= 1342], and it yields $r = 31_M \to t = 3_F$. The example shows that the algorithm set out at the beginning of this section is read (and should be read) as follows:

$(r,y) \to r + 3 + [\![(y \bmod 4)/4]\!] \to (r + 3 + [\![(y \bmod 4)/4]\!]) \bmod 31 = t.$

Comm. As Julian calendar years are modelled on the tropical year, the precession of the equinoxes is irrelevant to determining the date of Easter. Rhabdas' statement is therefore wrong. In sources contemporary with Rhabdas, Barlaam stressed a gap of about 1 day every 304 years between real and schematic Moons, based on a more accurate value of the mean synodic month (see sect. **7**). The gap accumulated since the times of the conception of the "table of the fathers", Barlaam writes, amounts to 2 days.[51]

[49] The latter is among the values used in Arabic astronomy; see the discussion in S. Mohammad Mozaffari, "A Medieval Bright Star Table: The Non-Ptolemaic Star Table in the *Īlkhānī Zīj*", *Journal for the History of Astronomy* 47 (2016), 294-316, in particular 303-307. Values such as $1^{\circ}/66^{y}$ were known to Byzantine astronomers acquainted with the Arabic tradition as early as *ca.* 1032: J. Mogenet, "Une scolie inédite du Vat. gr. 1594 sur les rapports entre l'astronomie arabe et Byzance", *Osiris* 14 (1962), 198-221, at 209 (section 29).

[50] On AD 1342 March 22, at 23:42 UT, according to http://www.eclipsewise.com/. Recall that the local time in Constantinople is nearly exactly UT + 2 hours and that a morning epoch was used in Byzantium, so that the entire night is attached to the previous day: Neugebauer, *HAMA* (cit. n. 29), 1069 n. 6.

[51] Tihon, "Barlaam" (cit. n. 20), 376-378 (sects. 23-29). The "table of the Fathers" denotes the Damascene table and its traditional list of Passover dates. John Damascenus lived about 600 years before Barlaam and Rhabdas.

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Τὴν δὲ ἐν τῷ θέρει μετὰ τὴν πεντηκοστὴν γινομένην Νηστείαν τῶν ἁγίων ἀποστόλων εὑρήσεις οὕτως. ἀρίθμησον ἀφ’ ἧς ἡμέρας ἐγένετο τὸ Πάσχα μέχρι τῆς τρίτης τοῦ Μαΐου μηνός, καὶ ὅσον ἀριθμὸν εὕρῃς, τοσαῦται εἰσὶ καὶ αἱ ἡμέραι τῆς ἐν τῷ θέρει γινομένης Νηστείας τῶν ἁγίων ἐνδόξων καὶ πανευφήμων ἀποστόλων.

Οἷον εὑρέθη τὸ εὐσεβὲς ἅγιον Πάσχα κατὰ τὴν λα τοῦ Μαρτίου μηνός, καὶ ἀριθμῶ ἀπὸ ταύτης μέχρι τῆς τρίτης τοῦ Μαΐου μηνός, καὶ εὑρίσκω ἡμέρας λγ, καὶ λέγω τοσαύτας εἶναι καὶ τὰς ἡμέρας τῆς ἐν τῷ θέρει νηστείας. ἵνα δὲ καὶ διὰ πλείονος βασάνου καὶ πείρας ἀληθὲς καὶ ἀναμφίβολον φανείη τὸ λεγόμενον, ἀρίθμησον ἀπὸ τῆς τοῦ Μαρτίου μηνός (ἤγουν ἀπὸ τῆς α τοῦ Ἀπριλλίου) τὰς ἐφεξῆς ἡμέρας τῶν μηνῶν μέχρι καὶ αὐτῆς τῆς κθ τοῦ Ἰουνίου μηνός, καθ’ ἣν ἡ σεβάσμιος τῶν ἁγίων ἀποστόλων γίνεται μνήμη, καὶ ἐξ αὐτῶν ἄφελε τῆς ἁγίας πεντηκοστῆς ἡμέρας ν καὶ ζ τῆς ἑβδομάδος τοῦ ἁγίου καὶ ζῳοποίου πνεύματος, καὶ αἱ καταλειφθεῖσαι εἰσὶν αἱ ἡμέραι τῆς Νηστείας[a] τῶν ἁγίων ἀποστόλων. εὑρίσκονται οὖν ἡμέραι ϙ οὕτως· λ τοῦ Ἀπριλλίου, λα τοῦ Μαΐου καὶ κθ τοῦ Ἰουνίου· ὁμοῦ καὶ πάλιν ϙ· ἐξ αὐτῶν οὖν ἐκβαλὼν ἡμέρας νζ ἐναπελείφθησαν ἡμέραι λγ, ὅσαι δηλονότι εὑρέθησαν καὶ διὰ τῆς προτέρας μεθόδου. τοσαῦτα σοι καὶ περὶ τῆς τῶν πασχαλίων εἰδήσεως.

[a] pro verbo signum

Apostles’ Fast

You will find the Apostles’ Fast that occurs in Summer after Pentecost as follows. Reckon from the day on which Easter occurred up to the third of the month of May, and whatever number you find, so many are also the days of the Fast of the holy, glorious, and all-praiseworthy Apostles, which occurs in Summer.

For instance, the sacred holy Easter has been found on the 31st of the month of March, and I reckon from this <day> up to the third of the month of May, and I find 33 days, and I say that so many are also the days of the aestival fast. In order for that which is said to appear true and unambiguous by means of a multiple check and test too, reckon, from the month of March (namely, from April 1), the successive days of the months up to and including the 29th of the month of June, on which the venerable remembrance of the holy Apostles occurs, and remove from them 50 days of the holy Pentecost and 7 of the week of the Holy and Life-creating Spirit, and those which are left out are the days of the Apostles’ Fast. Then, 90 days are found as follows: 30 from April, 31 from May, and 29 from June; together, again, 90 too; then, casting 57 days aside from them 33 days are left out, as many, as is clear, as were also found by means of the previous algorithm. So much for you about the knowledge of the paschalia too.

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***Par*.** Apostles' Fast (Νηστεία τῶν ἁγίων ἀποστόλων) is the period f_H reckoned from Easter to May 3. The algorithm for finding f_H being r the date of Easter is as follows:

$(r) \rightarrow r \in M \rightarrow 3_{Ma} - r = f_H$.

A computation is carried out for current year AM 6850 [= 1342], and it yields $r = 31_M \rightarrow f_H$ = 33 days.

And this suffices for a comprehensive survey of Paschalia.

***Comm*.** The rationale behind choosing May 3 as the reference date for the subtraction is the following:[52] Apostles' Fast begins the second Monday after Pentecost, that is, 57 days after Easter (57 = the 50 days of Pentecost plus 1 week) and ends on June 28. However, an inclusive time interval that begins 57 days after Easter and ends on June 28 is equal to an inclusive time interval that begins on Easter and ends 57 days before June 28, that is, on May 2. Prescribing May 3 means that one must reckon (and not count)[53] the days from Easter and up to May 3.

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Ἐγὼ δὲ καὶ λίαν τεθαύμακα πῶς οἱ θεσπέσιοι καὶ θεόφοροι πατέρες ἡμῶν οἱ ἐν τῇ πρώτῃ συνελθόντες συνόδῳ τὴν τοῦ Πάσχα ἐκθέμενοι εὕρεσιν, ὅπως οὐκ ἐφρόντισαν διὰ μεθόδου τινὸς τὴν τῆς Ἀπόκρεω προηγεῖσθαι εὕρεσιν, ἀλλὰ τοὐναντίον μετὰ τὴν τοῦ Πάσχα δήλωσιν τὴν Ἀπόκρεω ἐθέσπισαν εὑρίσκεσθαι. ἐμοὶ δὲ καὶ τοῦτο ἐκπονῆσαι καὶ ἀναπληρῶσαι διὰ ἁγίων ἐκείνων εὐχῶν κεχάρισται, ἄξιον δὲ καὶ τὴν αἰτίαν ἐκθέσθαι δι' ἣν εἰς τὸν ἀγῶνα τοῦτον ἐμαυτὸν καθῆκα. φιλονεικοῦντι μοι ποτὲ μετά τινος Ἰουδαίου περὶ τῆς ἡμετέρας πίστεως ὡς ἔγκλημά τι καὶ τοῦτο ἐπήγαγεν ὅτι δῆθεν ἄνευ τοῦ νομικοῦ Φασκαλίου τὸ ἡμέτερον Πάσχα εὑρεῖν οὐ δυνάμεθα· ὅθεν διαπονηθεὶς περὶ τούτου μέθοδόν τινα θαυμασίαν ἐφεῦρον ἥτις χωρὶς τοῦ νομικοῦ Φασκαλίου τὸ ἡμέτερον εὐσεβὲς καὶ ἅγιον εὑρίσκεται Πάσχα, πλὴν οὐκ ἀντιστρόφως, ὥσπερ δι' ἐκείνου πρῶτον μὲν εὑρίσκουσα τὸ Πάσχα, εἶθ' οὕτως δι' αὐτοῦ τὴν Ἀπόκρεω καὶ τὴν τοῦ θέρους νηστείαν πρωθύστερον, ἀλλ' ἐνορδίνως πρῶτον μὲν τὴν Ἀπόκρεω, εἶτα τὸ Πάσχα καὶ εὐθὺς τὴν ἐν τῷ θέρει [[Νηστείαν]] γινομένην τῶν ἁγίων ἀποστόλων Νηστείαν.

Ἔχει δὲ οὕτως. κράτησον τὸν ἐνεστῶτα τῆς σελήνης θεμέλιον οἷος ἐστὶ κατὰ τὴν πρώτην τοῦ Ἰαννουαρίου μηνός, καὶ ἔπαρον ἀπὸ τῆς ἀρχῆς τοῦ τοιούτου Ἰαννουαρίου μηνὸς ἡμέρας ὅσας χρήζεις εἰς ἐκπλή|68vρωσιν ἡμερῶν ν, συντιθεμένων δηλαδὴ μετὰ τοῦ θεμελίου τῆς σελήνης· εἰ δ' οὐκ ἐξαρκέσουσιν ὅ τε θεμέλιος καὶ αἱ πᾶσαι τοῦ Ἰαννουαρίου

52 See *Anonymus* 1247, sect. 24, in Schissel, "Chronologischer" (cit. n. 30), 110; *Anonymus* 1256, sect. 10; Isaac Argyros, in *PG* XIX, 1305 and 1308; *Anonymus* 1377, sect. 9, in *PG* XIX, 1328-1329.

53 The difference between counting and reckoning is usually formulated as the difference between inclusive and exclusive reckoning.

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μηνὸς ἡμέραι, τὰς λειπούσας εἰς ἀπάρτισιν τῶν ν ἡμερῶν λαμβάνω ἀπὸ τοῦ Φευρουαρίου μηνός – πλὴν εἰδέναι σε καὶ τοῦτο δεῖ, ὡς ὅταν ἐξισοῦται ὁ θεμέλιος τῆς σελήνης ταῖς τοῦ Φευρουαρίου μηνὸς ἡμέραις, καὶ ἕνι κη ἢ κθ, ἐᾶν χρὴ παντελῶς τὸν τῆς σελήνης θεμέλιον, καὶ λαμβάνειν ἐκ μόνων τῶν δύο μηνῶν τοῦ τε Ἰαννουαρίου καὶ τοῦ Φευρουαρίου τὴν τῶν ν ἡμερῶν ποσότητα – καὶ τὴν ποσταίαν τοῦ μηνὸς ἐκείνου ἐν ᾗ τυχὸν ἡ πεντηκοστὴ τῶν ἡμερῶν ἐτελεύτησεν εὑρὲ διὰ τῆς μεθόδου τοῦ ἡμεροευρεσίου ποία ἡμέρα τῆς ἑβδομάδος ἐστί, καὶ εἰ μὲν μία καταλειφθῇ, λέγω ταύτην εἶναι τὴν τοῦ Ἀσώτου κυριακήν, εἰ δὲ β, τὴν δευτέραν τῆς ἑβδομάδος τῆς Ἀπόκρεω, εἰ δὲ τρεῖς, τὴν τρίτην τῆς αὐτῆς ἑβδομάδος, καὶ καθεξῆς ὁμοίως μέχρι τοῦ σαββάτου, καὶ ἡ ἐρχομένη κυριακὴ τῆς αὐτῆς ἑβδομάδος δῆλον ὅτι ἐστὶν ἡ Ἀπόκρεω. κράτησον οὖν τὴν ποστὴν τοῦ μηνὸς ἐν ᾗ ἡ πεντηκοστὴ ἔτυχεν ἡμέρα· πρόσθες αὐτῇ καὶ τὰς ἐπιλοίπους ἡμέρας ἄχρι καὶ τῆς ἐρχομένης κυριακῆς, κἀκείνην λέγε εἶναι τὴν ἡμέραν τῆς Ἀπόκρεω. – ἔκτοτε οὖν εἰ βούλει γινώσκειν καὶ τὸ νομικὸν πότε γίνεται Φάσκα, σκόπει τὸν τῆς σελήνης κύκλον ὁποῖος ἐστί, καὶ εὑρήσεις ἐν τῇ ὑποκειμένῃ σοι κάτωθεν πλινθίδι τὸν παρακείμενον ἀριθμόν, εἴτε ἐν τῷ Μαρτίῳ ἐστὶν εἴτε ἐν τῷ Ἀπριλλίῳ.

Ἢ καὶ ἄλλως. ἐξέτασον τοῦ παρελθόντος χρόνου τὸ νομικὸν ποῦ ἔτυχε, κἂν μὲν εὕρῃς ὅτι εἰς τὸν Ἀπρίλλιον ἐγένετο, ὀπισθοπόδησον ἡμέρας ια, καὶ εἰς τὴν ἑξῆς, ἤτοι τὴν δωδεκάτην, ἴσθι εἶναι τὸ νομικόν· εἰ δὲ εἰς τὸν Μάρτιον ἔτυχε, πρόσθες νυχθήμερα ιη, καὶ εἰς τὸ ἐφεξῆς, ἤτοι τὸ ἐννεακαιδέκατον, γίνωσκε γίνεσθαι τὸ νομικόν. – εἶτα ἀρίθμησον ἀπὸ τῆς κυριακῆς τῆς Ἀπόκρεω ἡμέρας νς, καὶ ἔνθα ἂν καταλήξῃς, ἐκεῖ ἐστὶ τὸ Πάσχα· καὶ πάλιν ἀπὸ τοῦ Πάσχα μέχρι τῆς τρίτης τοῦ Μαΐου μετρῶν εὑρήσεις καὶ τὴν Νηστείαν τῶν ἁγίων ἀποστόλων.

Ἵνα δὲ καὶ μεθ᾽ ὑποδείγματος σαφέστερον γένηται τὸ λεγόμενον, ὑποκείσθω εὑρεῖν ἡμᾶς κατὰ τὸ νῦν ἐνεστὸς ͵ςων ἔτος τήν τε Ἀπόκρεω, τὸ νομικόν, τὸ εὐσεβὲς ἅγιον Πάσχα καὶ τὴν ἐν τῷ θέρει νηστείαν. καὶ ἐπειδὴ κατὰ τὸ τοιοῦτον ἔτος ὁ τῆς σελήνης κύκλος εὑρέθη δέκατος ὁ δὲ ταύτης θεμέλιος ἐστὶν εἰκοστότριτον, κρατῶ τοῦτον (ἤγουν κγ), καὶ λαμβάνω καὶ ἀπὸ τοῦ Ἰαννουαρίου εἰς ἐκπλήρωσιν τῶν ν ἡμέρας κζ. καὶ ἐπειδὴ εἰς τὰς κζ τοῦ Ἰαννουαρίου ἔληξεν ἡ πεντηκοστὴ τῶν ἡμερῶν, ἀναζητῶ διὰ τῆς μεθόδου τοῦ ἡμεροευρεσίου ποία ἡμέρα τῆς ἑβδομάδος τυγχάνει, καὶ εὑρίσκω ταύτην κατὰ τὴν προγραφεῖσαν μέθοδον οὕτως. ὁ τοῦ ἡλίου κύκλος ὑπάρχει ιηος· τούτῳ προστίθημι καὶ τὰ ἐπιβάλλοντα αὐτῷ ἀπὸ τοῦ βισέξτου τέταρτα, ἅπερ εἰσὶ δ· καὶ γίνονται κβ· ὁμοίως προστίθημι ταύταις καὶ τὰς ἐπακτὰς τῶν παρελθόντων τριῶν μηνῶν ἀπ᾽ ἀρχῆς Ὀκτωβρίου μέχρις Ἰαννουαρίου, αἳ καὶ εἰσὶν η· καὶ γίνονται μετὰ τῶν κβ, λ· ταύταις συντίθημι καὶ τὰς κζ τοῦ Ἰαννουαρίου· καὶ γίνονται πᾶσαι ὁμοῦ νζ, ἐξ ὧν ἀφαιρῶ ἑβδομάδας ὀκτώ· καὶ ἐναπελείφθη μοι ἡμέρα μία, ἥντινα καὶ λέγω εἶναι τὴν τοῦ Ἀσώτου κυριακήν, καὶ ἡ ἐρχομένη ἑτέρα κυριακή, ἥτις ἐστὶν ἡ τρίτη τοῦ Φευρουαρίου μηνός, ἐστὶν ἡ Ἀπόκρεω· ζ γὰρ καὶ κζ· γίνονται λδ, ἀφ᾽ ὧν ἔκβαλον τὴν λα τοῦ Ἰαννουαρίου· καὶ κατελείφθησαν ἡμέραι

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γ, αἵ καὶ εἰσὶ τοῦ Φευρουαρίου, εἰς ἣν δηλονότι λέγω εὑρεθῆναι καὶ τὴν Ἀπόκρεω. ζητῶ τὸ νομικόν, καὶ εὑρίσκω ἐν τῇ ὑποτεταγμένῃ πλινθίδι τῷ δεκάτῳ κύκλῳ τῆς σελήνης παρακειμένην τὴν κδ τοῦ Μαρτίου μηνός. ἢ καὶ ἄλλως. ἐξετάζω κατὰ τὸν παρελθόντα χρόνον ποῦ ἔτυχε τὸ νομικόν, καὶ εὑρίσκω τοῦτο εἰς τὰς δ τοῦ Ἀπριλλίου· ὀπισθοποδῶ ἐξ αὐτῆς ἡμέρας ια, καὶ καταντῶ καὶ οὕτως εἰς τὰς κδ τοῦ Μαρτίου |69r εἰς ἡμέραν κυριακήν, ὡς ψηφίζων εὑρήσεις.

Θέλω τὸ ἡμέτερον τῶν πιστῶν ἅγιον Πάσχα εὑρεῖν, καὶ ἀριθμῶ ἀπὸ τῆς κυριακῆς τῆς Ἀπόκρεω τὰς ἐφεξῆς ἡμέρας ἄχρις ἂν σώσω ἡμέρας νς, καὶ ὅπου συντελεσθῶσιν, ἐν ἐκείνῃ τῇ ἡμέρᾳ λέγω γίνεσθαι καὶ τὸ ἡμέτερον Πάσχα. οἷον εὑρέθη ἡ Ἀπόκρεω εἰς τὰς γ τοῦ Φευρουαρίου· καὶ ἐκβάλλω ταύτας ἀπὸ τῶν κη τοῦ Φευρουαρίου ἡμερῶν· ἐναπελείφθησαν καὶ ἡμέραι κε· ταύταις προστίθημι καὶ τὴν λα τοῦ Μαρτίου· καὶ γίνονται ὁμοῦ ἡμέραι νς. καὶ λέγω εὑρεθῆναι καὶ τὸ ἅγιον Πάσχα εἰς τὴν λα τοῦ Μαρτίου μηνός.

Ἢ καὶ ἄλλως. ἄφελε ἀπὸ τοῦ ἀριθμοῦ ἐν ᾗ εὑρέθη ἡ Ἀπόκρεω ἡμέρας γ, εἰ δὲ ἔστι βίσεξτον[a], δ, καὶ τὸν καταλειφθέντα ὅρα πόσος ἐστὶ καὶ ποῦ ἔτυχεν, ἤγουν εἰς τὸν Ἰαννουάριον ἢ εἰς τὸν Φευρουάριον, καὶ εἰ μὲν ἔτυχεν ἡ ἐναπολειφθεῖσα ποσότης μετὰ τὴν τῶν τριῶν ἢ τῶν δ ἀφαίρεσιν εἰς τὸν Φευρουάριον, λέγε κατὰ τήνδε γενέσθαι καὶ τὸ Πάσχα εἰς τὸν Ἀπρίλλιον· εἰ δὲ εἰς τὸν Ἰαννουάριον, λέγε εὑρεθῆναι εἰς τὸν Μάρτιον. οἷον εὑρέθη ἡ Ἀπόκρεω εἰς τὰς γ τοῦ Φευρουαρίου· καὶ ἀφελῶ ταύτας ἐπεὶ βίσεξτον[b] οὐκ ἔστι, καὶ καταλήγω εἰς τὴν λα τοῦ Ἰαννουαρίου, καὶ λέγω γενέσθαι τὸ Πάσχα εἰς τὴν λα τοῦ Μαρτίου μηνός.

Μετρῶ ἀπὸ ταύτης τὰς ἐφεξῆς ἡμέρας τοῦ Ἀπριλλίου μέχρι τῆς τρίτης τοῦ Μαΐου μηνός, καὶ εὑρίσκω ταύτας ἡμέρας λδ, καὶ λέγω εἶναι καὶ τὰς ἡμέρας τῆς ἐν τῷ θέρει Νηστείας τῶν ἁγίων ἀποστόλων ἡμέρας λδ.

Ἔχε καὶ ταύτην τὴν μέθοδον μὴ διαφεύγουσάν σου τὴν ἔφεσιν, φίλων ἄριστε καὶ ἐράσμιε.

α	Ἀπρ	β	β	Μάρ	κβ	γ	Ἀπρ	ι	δ	Μάρ	λ	ε	Ἀπρ	ιη	ς	Ἀπρ	ζ
ζ	Μάρ	κζ	η	Ἀπρ	ιε	θ	Ἀπρ	δ	ι	Μάρ	κδ	ια	Ἀπρ	ιβ	ιβ	Ἀπρ	α
ιγ	Μάρ	κα	ιδ	Ἀπρ	θ	ιε	Μάρ	κθ	ις	Ἀπρ	ιζ	ιζ	Ἀπρ	ε	ιη	Μάρ	κε
ιθ	Ἀπρ	ιγ	κατ	ὰ λή	θην ἐ	γένο	ντο	τὰ π	ολλ	ὰ τε	τρά	γων	α				

[a] lege βίσεξτος [b] lege βίσεξτος

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A paschalion without using Passover

I have always been utterly surprised by the fact that our divine and god-carrying Fathers, those that met in the first synod to set out the finding of Easter, I mean, by the fact that they did not put their minds to make, by whatever algorithm, the finding of Meat-Fare come first, but, on the contrary, they decreed that Meat-Fare should be found after the explanation of Easter. It has been a pleasure for me to work hard on this too, and to succeed thanks to such-and-such holy prayers; still, it is also worth setting out the reason why I resolved to get involved in this game. For I was once engaged in a discussion with a Jew about our faith; he also adduced this as a charge of sorts, that betcha we are unable to find our Easter without Passover; whence, working hard on this I found out a wonderful algorithm that can find our sacred and holy Easter independently of Passover, except that <this does not happen> in perturbed order, as for <an algorithm> that first finds Easter by means of that one [*scil.* Passover], then accordingly Meat-Fare by means of it [*scil.* Easter], and the aestival fast first last, but in due order, Meat-Fare first, then Easter, and forthwith the Apostles' Fast that occurs in Summer.

It is as follows. Keep the present cycle of the Moon what<ever> it is on the first of the month of January, and, from the beginning of such a month of January, raise as many days as you need for the filling of 50 days, compounded, as is clear, with the base of the Moon; if both the base and all days of the month of January do not suffice, I take the <days> missing for the completion of the 50 days from the month of February—except that you must also know this, that, whenever the base of the Moon is equal to the days of the month of February, and this is 28 or 29, one has to neglect the base of the Moon altogether, and to take the quantity of 50 days from the two months of January and February only—and find, by the day-finding algorithm, what weekday is the date of that month in which the fiftieth day ended, and if one <day> is left out, I say that this is the Sunday of the Prodigal Son, if 2, it is Monday of the week of Meat-Fare, if three, it is Tuesday of the same week, and similarly in succession up to Saturday, and it is clear that the forthcoming Sunday of the same week is Meat-Fare.[54] Then, keep the date of the month on which the fiftieth day falls; add to it the remaining days up to and including the forthcoming Sunday too, and say that that one is the day of Meat-Fare. — Then, thereafter, if you also wish to know when Passover occurs, consider the cycle of the Moon what<ever> it is, and you will find, in the table set out below for you, the number that is placed next <to it>, whether it is in March or in April.

Par. Rhabdas wonders why the Church Fathers chose to determine the date of Meat-Fare Sunday after the date of Easter rather than vice versa. Spurred by an exchange with a Jew—who

[54] As Sunday is the first day of the week, this statement is inaccurate.

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reproached Christians for not being able to compute the date of Easter without using the date of Passover—Rhabdas set out to find an algorithm for computing the dates of Meat-Fare Sunday and of Easter, and the duration of Apostles' Fast, in this order and without computing the date of Passover.

The algorithm for finding such a paschalion once the base b_m is given [the sign $S(*)$ is the Sunday next after day *] is as follows:

$(b_m) \rightarrow 50 - b_m -: 1_J \rightarrow w(50 - b_m -: 1_J) \rightarrow S[w(50 - b_m -: 1_J)] = f_m \rightarrow f_m + 56 = r_m \rightarrow 3_{Ma} - r_m = f_{H,m}$.

Comm. Rhabdas reuses a part of his *Letter to Tzavoukhes*.[55] To explain his rule, I shall combine the definition of "base" given in sect. **6** (which, contrary to what Rhabdas claims, is not the "base on January first"): $b_m = (11m + 3) \bmod 30$, and the rule for Passover given in sect. **10**: $p_m = 50 - [(11m + 6) \bmod 30] -: 1_M$. Bypassing for the sake of simplicity some niceties of modular reduction, we get $p_m = 50 - 3 - [(11m + 3) \bmod 30] -: 1_M = 50 - 3 - b_m -: 1_M = 47 - b_m -: 1_M$. Now, counting lunar days starting from January 1 rather than March 1 adds 59 days: $p_m = 59 + 47 - b_m -: 1_J = 106 - b_m -: 1_J$. We are seeking a proxy for Passover such that Meat-Fare Sunday is the proxy for Easter: therefore, the proxy for Passover must precede Passover by 8 weeks, that is, by 56 days: $Proxy(p_m) = 106 - 56 - b_m -: 1_J = 50 - b_m -: 1_J$. Rhabdas asserts that values of $b_m = 28, 29$ (that is, $m = 5, 16$) must be disregarded, counting directly 50 days from January 1. The reason is that January 21 or 22 are too early dates for a proxy Passover, for Passover would thus fall on March 19 or 20 at the latest. The striking anomaly of this algorithm is that the *saltus lunae* disappears, unless it is included in the definition of the base, which is not what Rhabdas does. This—and the mistake in locating the *saltus* itself in the main algorithm found in sect. **10**—shows that Rhabdas drew this idea and most of his material from some previous authority, and incorporated this information into his text without fully understanding its implications.

To my knowledge, Rhabdas is the first author who sets forth, and openly presents it as such, an algorithm for computing the date of Easter without mentioning Passover. However, the same algorithm, without Rhabdas' sagacious point about the algorithm not using Passover, is found in the 1335 Computus contained in Matthew Blastares' Σύνταγμα.[56] As Blastares' treatise is a compilation, it is likely that Rhabdas and Blastares depend on a common source. Isaac Argyros appropriated the same idea in his Computus dated 1372 and claimed that it was his own discovery, which is exactly what Rhabdas had claimed thirty-one years before.[57]

[55] Tannery, "Notice" (cit. n. 15), 134.23-138.27, both verbatim and after rewriting.

[56] Rhalles, Potles, Σύνταγμα (cit. n. 30), 418-419.

[57] See the discussion in O. Schissel, "Die Osterrechnung des Nikolaos Artabasdos Rhabdas", *Byzantinisch-neugriechische Jahrbücher* 14 (1938), 43-59.

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An alternative algorithm for computing Passover; computation of a paschalion

Or also in another way. Ascertain where Passover fell in the bygone year, and if you find that it occurred in April, walk 11 days back, and know that Passover is on the successive <day>, viz. the twelfth; if <Passover> fell in March, add 18 nychthemera, and know that Passover occurs on the successive <nychthemeron>, viz. the nineteenth. — Then reckon 56 days from Meat-Fare Sunday, and wherever you stop on, Easter is there; and again, by determining <the number> from Easter up to the third of May you will also find Apostles' Fast.

In order for that which is said to be also clearer with an example, let it be supposed for us to find, in the now-current year 6850, Meat-Fare, Passover, the sacred holy Easter, and the aestival fast. And since in such a year the cycle of the Moon has been found to be the tenth and its base is the twenty-third, I keep this (namely, 23), and I also take 27 days from January for filling 50. And since the fiftieth day stopped on January 27, by means of the day-finding algorithm I seek what weekday happens to be, and I find this by means of the algorithm written above, as follows. The cycle of the Sun turns out to be the 18th; I also add to this the fourths cast upon it from the leap day, which are 4; and they yield 22; similarly I also add to these the epacts of the three months bygone from the beginning of October up to January, which are also 8; and, with 22, they yield 30; I also compound the 27 <days> of January with these; and all of them together yield 57, from which I remove eight weeks; and one day is left out for me, which I also say to be the Sunday of the Prodigal Son, and the other, forthcoming Sunday, which is the third of the month of February, is Meat-Fare, for 7 and 27; they yield 34, from which cast aside 31 <days> of January; 3 days are also left out, which are also in February, on which <day>, as is clear, I say that Meat-Fare has also been found. I seek Passover, and I find, in the table arranged below, that the 24th of the month of March is placed next to the tenth cycle of the Moon. Or also in another way. I ascertain where Passover fell in the bygone year, and I find this on April 4; I walk 11 days back from it, and in this way too I arrive at March 24, a Sunday, as you will find by calculation.

I want to find our holy Easter of the believers, and I reckon, from Meat-Fare Sunday, the successive days until I have enough for 56 days, and wherever they are completed, I say that on that day our Easter also occurs. For instance, Meat-Fare has been found on February 3; and I cast these aside from the 28 days of February; 25 days are also left out; I also add the 31 <days> of March to these; and together they yield 56 days. And I say that the holy Easter has also been found on the 31st of the month of March.

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Par. The date of Passover for each lunar cycle year is set out in a table given at the end of the treatise. An alternative algorithm, for computing Passover at lunar cycle year $m + 1$ once Passover at year m is known, is as follows:

$(p_m) \rightarrow$

$| \, p_m \in A, p_m - 11 = p_{m+1}.$

$| \, p_m \in M, p_m + 19 = p_{m+1}.$

Comm. This algorithm formalizes the following data:[58] since each year the epacts increase by 11 units, the date of Passover shifts backwards by 11 days from an assigned year to the next. However, Passover cannot fall earlier than March 21; therefore, such early dates are replaced by a date falling one lunar month later. Given the fact that $19 \equiv -11 \pmod{30}$ and that Rhabdas does not use monthly dates for p_{m+1} but counts calendar days from March 1, whenever the Passover date falls outside the lower bound, March 21, of the 30-day Passover interval $21_M \leq p \leq 19_A$, it enters again this interval from its upper bound, April 19, increased by 19 days.[59] As usual, Rhabdas makes a mess of the figures by mixing up counting and reckoning; he does not take into account the *saltus lunae*, either.

An alternative algorithm for computing Easter

Or also in another way. Remove 3 days from the numerical date on which Meat-Fare has been found, 4 if it is a leap year, and look at how much it is and where it falls, namely, whether in January or in February, the <numerical date> left out, and, if the quantity left out after the removal of three or of 4 falls in February, say that, according to this, Easter also occurs in April; if, however, <the former falls> in January, say that <the latter> has been found in March. For instance, Meat-Fare has been found on February 3; and I remove these because it is not a leap year, and I stop on January 31, and I say that Easter occurs on the 31st of the month of March.

Starting from this, I determine the successive days of April up to the third of the month of May, and I find that these days are 34, and I say that 34 days are also the days of the aestival Apostles' Fast.

Pay also attention that this algorithm will not escape you, best and loveliest of my friends.

[58] Similar algorithms are found, for instance, in George, sect. 3, in Diekamp, "Der Mönch" (cit. n. 34), 29.7-30.2; *Anonymus* 892, sect. 26; Psellos, sect. I.4, in Redl, "La chronologie" (cit. n. 36), I, 213-215; Matthew Blastares, in Rhalles, Potles, Σύνταγμα (cit. n. 30), 417 n. 1.

[59] This rule shows that the actual terms for Passover are $21_M \leq p \leq 19_A$; the traditional terms $21_M \leq p \leq 18_A$ discard April 19 because this date coincides with a gap of the Passover sequence and it is located at the end of the interval (see note 22 above).

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Par. A computation is carried out for current year AM 6850 [= 1342], and it yields (m = 10, $s = 18$, $p_m = 24_M$)[60] $\rightarrow f_m = 3_F \rightarrow r_m = 31_M \rightarrow f_H = 34$ (the right value is 33; Rhabdas counted days instead of reckoning them, see sect. **14**).

An alternative algorithm for Easter is:

$(f) \rightarrow f - (3 + [\![(y \bmod 4)/4]\!]) -: 1_{J,F} = r -: 1_{M,A}$.

A Passover table is finally provided, originally set out with too many cells, as a phrase written within the table itself confirms ("too many squares resulted because of a mistake"):

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
p_m	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17	5	25_M	13

Comm. The table sets out the traditional Passover dates. The table also shows that, in the above rule for computing Passover at lunar cycle year $m + 1$ once Passover at year m is known, Rhabdas neglects the *saltus lunae*, for p_{17} and p_{16} differ by 12 days, not by 11 days as is prescribed by the rule.

Appendix. A Thematic Word Index to Rhabdas' Computus[61]

Chronological terms

A "cycle" (κύκλος: 2-6, 10-12, 15) first "begins" (ἄρχεται: 3, 5), then "reaches to" (ἀνέρχεται εἰς: 3, 5) its last year, and finally "takes again its beginning" / "begins again first" (πάλιν λαμβάνει ἀρχήν / ἄρχεται πρῶτος: 3, 5). Temporal segments and computations go "from" (ἀπό; for instance the "years from the foundation of the world" ἀπὸ κτίσεως κόσμου ἔτη: 2, 4, 5, 12) the first item "up to" (μέχρι / ἄχρι: 2, 7, 9-11, 13-15) the last one, which is included if καί (translated "and including": 7, 13-15) is added. Dates can be "above" (ἄνωθεν; also ἔσωθεν "within": both in 12, or ὑπερβάλλων "exceeding": 13) or "below" (κάτωθεν) an assigned date (12). Past time segments are "bygone" (παρελθόντες: 7, 10-11,

[60] As the clever algorithm given at the beginning of this section computes the date of Easter without using the date of Passover, the latter date is read in the table attached to the end of Rhabdas' Computus or it is computed by means of the last algorithm.

[61] The most important computistical terms were also given above, in the commentary to the relevant sections of Rhabdas' Computus. After each lexical item, the sections of Rhabdas' Computus that contain it are indicated; I skip the boldface. Here I adopt the same translations I give in the thematic word index in Acerbi, "Byzantine Easter Computi" (cit. n. 19). For the partly overlapping technical lexicon of *Rechenbücher*, see K. Vogel, *Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts* (Wiener Byzantinische Studien 6), Wien 1968, 141-143, and the thematic word index in F. Acerbi, "Byzantine *Rechenbücher*: An Overview with an Edition of *Anonymi* L and J", *Jahrbuch der Österreichischen Byzantinistik* 69 (2019), 1-57, at 17-20.

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15); the current (cycle) year and the "base" (θεμέλιος: 1, 3, 6-8, 15) are "present" (ἐνιστάμενος: 2; ἐνεστώς: 6, 8, 10, 15; παρών: 6, 10, 12) "now" (νῦν: 2, 6, 10-12, 15); days next in a sequence are "forthcoming" (ἐρχόμεναι: 8, 11, 15) or "subsequent" (ἐπιοῦσαι: 8, 11); a structureless time-token is an "occasion" (καιρός: 2, 9, 10, 13). The first "day" (ἡμέρα: 6-15) of a "month" (μήν: 2-3, 5, 7, 8, 10-15) is its "beginning" (ἀρχή: 11, 15); the "year" is ἔτος (2, 4-6, 10, 12, 15), ἐνιαυτός (2, 11-13), or χρόνος (2, 3, 5, 8-10, 12, 13, 15). The determination of the date on which a festival someone "celebrates" (ἑορτάζει: 11) "falls" (τυγχάνει: 11, 13, 15) or "occurs" (γίνεται: 11, 13-15)—or which "receives" (ὑποδέχεται: 11) it or on which "it is celebrated" ([ἐπι– / συν]τελεῖται: 11, 13, 15)—is stressed by the correlatives "where[ver] … there" (ἔνθα … ἐν ταύτῃ / ἐκεῖ: 10 / 15) and "wherever … in that day" (ὅπου … ἐν ἐκείνῃ τῇ ἡμέρᾳ: 15); this date may "overrun" (παρατρέχειν: 13) an expected date. The day of the "week" (ἑβδομάς: 11, 13-15) on which a date falls is found by means of a "day-finding" (ἡμεροευρέσιος: 11, 15) algorithm. The "age of the Moon" (ἡ ἡμέρα τῆς σελήνης: 7-9; τὸ πόσων ἡμερῶν ἐστὶν ἡ σελήνη: 7, 8; the "Moon" [σελήνη: 2-3, 5-10, 13, 15] is also called μῆνις: 9) or a "date" (ποστή: 7, 11, 13, 15; ποσταία: 15) that "falls" (ἐμπίπτει: 13) within a month is also called "quantity" (ποσότης: 7, 13, 15); the "lunar month" is also called φεγγάριος (8). The Moon "shines" (φαίνει: 9; φαύει: 9; λάμπει: 8, 9; μαρμαίρει: 9; or δᾳδουχεῖ: 8 [here translated "to carry the torch alight"], 9) so many "hours" (ὧραι: 8, 9, 12) and "minutes" (λεπτά: 7-9) in a "night" (νύξ: 9). A 24-hour day is a "nychthemeron" (νυχθήμερον: 9, 12, 15), which in specific conditions is "completed" (ἀποτελούμενον: 12). The "Sun" (ἥλιος: 2-4, 8, 11, 12, 15) completes a year in 365 or 366 days, the latter occurring in a "leap year" (βίσεξτος: 8, 11-13, 15; in 8, 11, and 12 also "leap day"; in 12 "bissextile", as an adjective). The "epacts" (ἐπακταί: 8, 11, 15) and the "indiction" (ἴνδικτος: 2, 8) also belong to this lexical domain.

Specific mathematical terms

Investigation. γινώσκω: to know (15); γνωρίζω: to recognize (9, 13); ἐξετάζω: to ascertain (15); εὑρίσκω: to find (2, 4-15) and εὕρεσις: finding (2, 12); (ἀνα)ζητέω: to seek (1, 8, 9, 12-13, 15) and ζήτησις: search (7, 9-11); κατάληψις: apprehension (2); μανθάνω: to learn (1, 9, 11, 13), and μάθησις: learning (1); ὁράω: to look at (15); σκοπέω: to consider (2, 15); ὑπόκειμαι: to suppose (15).

Initializing an algorithm. κρατέω: to keep (2, 4-8, 11, 15).

Counting and reckoning. ἀπαρτίζω: to complete (12) and ἀπάρτισις: completion (15); ἀριθμέω: to reckon (14, 15) and ἀριθμός: number (2, 6, 7, 9-11, 13-15); ἐξισοῦμαι: to be equal to (8, 15); καταντάω: to arrive at (15); κατέχω: to hold (8; "to collect", said of taxes: 2); κρατέω: to keep (10); (κατα)λαμβάνω: to take (6, 8, 10, 11, 15); (κατα)λήγω: to stop on (12, 15); λογίζομαι: to be reckoned (9); μετρέω: to determine (11, 15); ὀπισθοποδέω:

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to walk back (15); πληρόω: to fill (10) and ἐκπλήρωσις: filling (10, 15); ποσόω: to amount to (7, 11); τελευτάω: to end in (12, 15); ὑπερβαίνω: to overstep (8, 13); φθάνω: to happen … first (10); ψηφίζω: to calculate (15) and ψῆφος: calculation (13). The result of any operation is indicated by ποιέω: to make (8, 9, 11). Any quantity can be a "whole" (ὁλόκληρος: 11) and possibly a "part" (μέρος: 9), that is, a fraction.

Identification of the result of an operation as a chronological item. ἀποφαίνομαι: to declare (9); γινώσκω: to know (15); εὑρίσκω: to find (13, 15); λέγω: to say (6, 8-12, 14, 15); νοέω: to consider (7), to conceive (9); οἶδα: to know (13, 15).

Unknown quantities. οἷος: what<ever> (6, 9-11, 14, 15); ὁποῖος: what (11, 15); ὅπου: wherever (15); ὅσος: as many, how many, whatever (6, 7, 9, 10, 14, 15); ὁσάκις: how many times (4); ποῖος: what (11, 13, 15); ποσάκις: how many times (2); πόσος: what (15), how much (7-9); πόστος: what (in an ordered sequence) (13); ποῦ: where (15); τοιόσδε: such (11); τοιοῦτος: such (2, 7, 10, 12, 13, 15); τοσοῦτος: such, so much (2, 8, 9, 12, 14).

Numerical sets. ἑβδομάς: week (11, 13-15); ἑξηκοντάς: sixty (7, 8); πεντάς: pentad (9); πεντεκαιδεκάς: pentadecad (2, 9, 19); τετράς: tetrad (12); τριακοντάς: thirty (6-8, 10), χιλιάς: thousand (10).

Operations

Addition. (συν)ἀθροίζω: to put together (7, 10); ἐπιβάλλω: to cast upon (9, 11, 15); ἑνόω ὁμοῦ: to unite together (11); προστίθημι: to add (6-8, 10-13, 15); συνάγω: to gather (7-9, 11); συντίθημι: to compound (15). The result is indicated by γίνομαι: to yield (2, 4-6, 8-10, 13, 15); ὁμοῦ: together (5-6, 8, 10, 11, 13-15); συναγωγή: gathering (2). The operation is called προσθήκη: addition (8).

Subtraction. ἀφαιρέω: to remove (2, 4-11, 13-15); ἐκβάλλω: to cast aside (2, 4, 9, 13-15); ἐπαίρω: to raise (10, 15). The remainder is indicated by the following items: ἐναπολείπομαι (2, 4, 5, 8, 10-15), καταλείπομαι (7, 8, 11-15), and καταλιμπάνομαι (2, 5, 6): to be left out; λοιπός (10) and ἐπίλοιπος (10, 15): remaining; predicative λοιπά: as a remainder (2, 4, 5); μένω: to remain (2, 4, 5, 9); περιττεύω: to remain over (2, 8). The operation is called ἀφαίρεσις: removal (15).

Multiplication. πολυ– / πολλαπλασιάζω: to multiply (9); μετρέω: to measure (10). However, multiplication is mainly formulated by means of an –ακις adverb (2, 6, 9, 10, 12). The result is indicated by participial forms of γίνομαι: to result (6, 9). The operation is called πολλαπλασιασμός (6, 10).

Taking multiples. ἑνδεκαπλασίασον: undecuple (6, 10); τετραπλασίαζε: quadruple (9).

Division. μερίζω παρά: to divide by (4, 5, 9),

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Modulo reduction. ἀναλύω εἰς / ἐπί / παρά: to resolve out into (2, 5, 12); ἀφαιρέω παρά: to remove by (4); διαιρέω εἰς: to remove by (12); ὑφαιρέω ἐπί: to remove by (4, 12). The remainder is indicated by the participial forms τὰ εὑρισκόμενα / καταλειφθέντα / ἐναπολειφθέντα κάτωθεν: that which is found / left down (2, 4-7, 10-12, 15).

Metadiscourse

Mathematical universality and generality are conveyed by the adverbs “always” (ἀεί: 9, 13; πάντως: 2, 4, 9, 12, 13; πάντοτε: 13) and “altogether” (παντελῶς: 13, 15). Iteration is formulated by “successive[ly]”, “in succession” ([καθ– / ἐφ]έξῆς: 7, 8, 11, 13-15), approximation by “very nearly” (ἔγγιστα: 8). An “algorithm” (μέθοδος: 1 [here translated “systematic exposition”], 2, 4, 5, 7-11, 14, 15; ἔφοδος: 9) may be “easy” (ῥᾴδιος: 2, 8, 12, 13), “concise” (σύντομος: 2, 4, 5, 8, 12), “general” (καθόλου: 8; καθολική: 7), and “exact” or “accurate” (ἀκριβής: 7, 13), it may also “prove true” ([ἐπ]αληθεύει: 9). A quantity to be discarded is marked by “to neglect” (ἐάω: 15). Examples are introduced by “for instance” (οἷον: 9, 14, 15) or by “for example” ([ἐπὶ / μεθ’] ὑποδείγματος [χάριν]: 6, 8-11, 15). Metamathematical markers include the modal operators “one must” (δεῖ: 10, 15) and “by necessity” (ἐξ ἀνάγκης: 8, 11), verbal adjectives with termination –τέον (2, 3, 8-13), the verb “to need” (χρήζω: 10, 11, 15), modal “should” (μέλλει: 8, 9, 11, 13), the volition verbs “to wish” (βούλομαι: 2, 4, 5, 15), “to want” ([ἐ]θέλω: 7, 9, 11, 12, 15), and “to hesitate” (ὀκνέω: 2). The verb forms “(we) say” (λέγομεν: 4, 6, 8-12; λέγε: 2; εἰπέ: 2, 5) and “(we) do” (ποίει: 4, 5; ποιοῦμεν: 10) introduce a computation. The adverb “as follows” (οὕτως: 2, 4-6, 8-15) introduces an algorithm; the adverb “similarly” (ὁμοίως: 11, 12, 15) replaces an algorithm that is identical to an algorithm previously carried out. The syntagms “as clarified / said above” (ὡς [ἀνωτέρω] δεδήλωται / εἴρηται: 2, 4, 5, 8, 12), “given” (δοθεῖσα: 9; δεδομένη: 10), “written above” (προγραφεῖσα: 15), and “expounded (above)” ([προρ]ρηθεῖσα: 9) refer to a known algorithm. An entry in a “table” (πλινθίς: 15) has a number “placed next” (παρακείμενος: 15) to it.